EDHEC BUSINESS SCHOOL

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EDHEC BUSINESS SCHOOL

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Contents

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Introduction

It is my pleasure to introduce the latest issue of the EDHEC Research Insights supplement to IPE, which aims to provide European institutional investors with an academic research perspective on the most relevant issues in the industry today.

Even though gaining explicit exposure to priced risk factors in the equity space is expected to provide good long-term risk-adjusted performance, investing in these factors also exposes the investor to a number of hidden or implicit risks that could be important drivers of short-term performance. In our article, we document and gain a better understanding of these hidden risks.

We assess the investability of smart beta equity strategies, as they naturally incur additional implementation hurdles compared to cap-weighted indices. While there are different dimensions related to investability, such as liquidity, capacity and transaction costs, it is possible to provide transparency on these dimensions with a range of metrics developed in market microstructure research. Our article introduces a suite of analytics to enable investors to assess the investability of smart beta indices.

Multi-factor index providers have been debating the respective merits of the 'top-down' and 'bottom-up' approaches to multi-factor equity portfolio construction. We review general insights from the literature on return estimation and factor models that are relevant for multi-factor portfolio construction and discuss recent literature that specifically addresses issues with bottom-up portfolio approaches.

An understanding of the design choices underlying multi-factor products is crucial if investors are to avoid outcomes that may ultimately disappoint them. Using evidence and beliefs, authors from Legal & General Investment Management (LGIM) outline a 'blank-sheet-of-paper' approach to designing a particular strategy that places a heavy emphasis on diversification at the factor, region, sector and stock level. This leads to considered objectives for portfolio return, risk and diversification that can be clearly messaged to investors.

In an article drawn from the Amundi "ETF, Indexing and Smart Beta Investment Strategies" research chair at EDHEC-Risk Institute, we clarify the various possible definitions of factors that are relevant in investment practice and develop a framework for allocating to factors in two main contexts, namely allocation decisions at the asset class level, and benchmarking decisions within a given class. It is possible to use factor indices as building blocks and to diversify risk across underlying factors, or to seek to exploit knowledge of economic regimes to design portfolios that react to changes in market conditions.

Goal-based investing principles can be used to effectively address the retirement investing problem by allowing investors in transition to secure minimum levels of replacement income for a fixed period of time in retirement, and also generate the kind of upside needed to reach target levels of replacement income with attractive probabilities. The emergence of the goal-based investing paradigm has effectively allowed for the development of mass-customised investment solutions to individuals. Risk management will play a central role in what should be regarded as nothing short of an industrial revolution that is impacting the investment management industry.

Being able to estimate the risk premium attached to Treasury bond yields in a reliable and robust manner is key to successful investing. EDHEC-Risk Institute has therefore launched the ERI Risk Premium Monitor: a robust tool to derive a state-of-the art estimation of the risk premium using market and monetary-policy information. Our article explains how this task is achieved and the theoretical underpinnings of the analytical tools used for the task.

We hope that the articles in the supplement will prove useful, informative and insightful. We wish you an enjoyable read and extend our warmest thanks to IPE for their collaboration on the supplement.

Noël Amenc, Associate Dean for Business Development, EDHEC Business School, CEO, ERI Scientific Beta

Misconceptions and mis-selling in smart beta:

Improving the risk conversation in the smart beta space

Eric Shirbini, Global Research and Investment Solutions Director, ERI Scientific Beta

Even though gaining explicit exposure to priced risk factors is expected to provide good long-term risk-adjusted performance, investing in these factors also exposes the investor to a number of hidden or implicit risks that could be an important driver of short-term performance.

Documenting such risk exposures is crucial to reconcile them with investors' preferences. With the calling into question of the default option that the cap-weighted index represented as a passive investment reference, smart beta's main fiduciary message is that there is no best solution in general, but instead a best solution that allows the investor's fiduciary choices to be executed in the most efficient way.

Ultimately, the choice on managing these risks is a key fiduciary decision that cannot be left to the appreciation of an index provider who has no status to do so.

Asset owners' governance practices should also be improved by starting a risk conversation on smart beta investments with stakeholders.

A recent EDHEC survey on equity factor investing (Amenc et al [2017a]) highlights that one of asset owners' primary motivations for investing in smart factors was to replace costly active managers with indices representative of a choice of factors that are well rewarded over the long term. In addition to providing access to well-rewarded risk factors at a lower cost compared to active managers, the asset owner also benefits from having much greater transparency and explicit control over which well-rewarded factors to invest in. Unfortunately, more often than not, the decision on which single or multi smart-factor index to invest in is made simply on the basis of the lowest cost and recent performance. However, by making an explicit choice over which factors to invest in, the asset owner now also takes on the fiduciary responsibility of its investment choices, which in the past had been delegated to the asset manager. Most of the performance and 'alpha' of asset managers came from implicit factor choices, as was well documented in Ang et al's report for the Norwegian Government Pension Fund in 2009. Investing in smart beta requires the asset owner to understand the consequences of making this investment decision. Even though gaining explicit exposure to priced risk factors (such as size, value, momentum, low volatility or quality) is expected to provide good long-term risk-adjusted performance, investing in these factors also exposes the investor to a number of hidden or implicit risks which could be an important driver of short-term performance. The three main categories of implicit risks are market risk bias, macroeconomic risks and sector/geographical risks.

The objective of this article is to document and gain a better understanding of these hidden risks. It is important to document these risks for three reasons.

Firstly, to avoid misinterpreting back-tested results, since short-term back-tests may be heavily influenced by the conditions prevailing over a particular period relating to these hidden risks. For example, the observed outperformance of a low volatility or defensive strategy may have been driven by falling interest rates and the sensitivity of low volatility stocks to changes in interest rates. This observation does not call into question the long-term reward from investing in such a strategy but does question whether this interest rate exposure is desirable or not and, of course, the limitations of investing in only a single factor.

Secondly, smart beta strategies may have an unexpected interaction with other asset classes included in the policy benchmark, leading to a potential misalignment with the objectives of the policy benchmark. One of the key conclusions of the seminal study by Ang et al (2009) of the Norwegian Government Pension Fund was to consider a framework that more explicitly recognises the structure of its return-generating process via investment in factor benchmark portfolios that go beyond asset classes. A better model of diversification is to diversify across these factors rather than relying on asset class diversification. In such a framework it is important to recognise an implicit risk such as exposure to interest rate risk, which may already be present in the fixed income part of the portfolio, by investing explicitly in a low volatility equity factor.

Lastly, unveiling these hidden or implicit risks also leads to better governance. A key question for investors is how to evaluate and communicate on the risks of their smart beta investments with respect to the different stakeholders. Ang and Kjaer (2011) note that "a thorough understanding by the asset owner of the key factor drivers of risk and return [...] is the best way to counter [governance problems]". An explicit choice of equity factors by asset owners naturally improves governance, compared to implicit choices made by active managers. However, equity factors themselves lead to other implicit risks, which need to be documented. For example, an explicit choice to get value exposure may lead to unintended macroeconomic risks (since value tends to do poorly when the term spread or industrial production declines and this type of macroeconomic risk is very different depending on the country). Popular minimum volatility strategies tend to result in countercyclical exposure because these strategies overweight healthcare and underweight energy stocks by about 5% compared to the market index. Just like monitoring the style drift of active managers, investors need to monitor the risk dynamics of factor strategies. In addition, a strategy with constant exposure to value may have time varying market betas, because, like the momentum factor, this strategy exhibits strong variations in market beta. These variations naturally have strong consequences on the out-of-sample performance of the strategy and more globally on their conditionality. This subject of the implicit market beta bet taken by factor strategies is probably one of the most poorly documented points in the academic literature devoted to factors, as shown by Amenc et al (2018).

Mind the market beta gap

The primary focus of the vast majority of providers of multi-factor strategies is on improving factor intensity in the hope of benefiting from higher premia in the long term. There is much debate on the best way to improve the performance associated with these factor exposures. In the same way, the question of improving factor intensity in the case of multi-factor assemblies has given rise to extensive literature. Amenc et al (2017b, 2017c) published two important contributions on this subject in 2017. However, little attention is paid to the management of market beta in such multi-factor strategies. This may be surprising because, while it is obviously

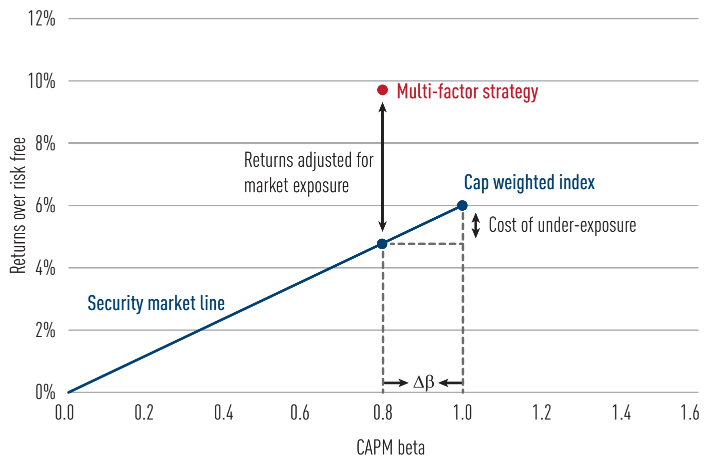
1. Impact of market beta on performance

Factor exposure	
Annualised unexplained	**1.92%**
Market beta	**0.97**
SMB beta	**0.18**
HML beta	**0.14**
MOM beta	**0.04**
Low volatility beta	**0.14**
High profitability beta	**0.09**
Low investment beta	**0.07**

Performance attribution	
Annualised unexplained	2.99%
Market factor	**5.71%**
SMB factor	0.34%
HML factor	0.01%
MOM factor	0.17%
Low volatility factor	−0.22%
High profitability factor	0.19%
Low investment factor	0.18%

Universe is EDHEC Risk US Long Term Track Records. Time period of analysis is from 31 December 1975 to 31 December 2015 (40 years). The analysis is based on weekly total returns in US dollars. All statistics are annualised. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. The market factor is the excess return series of the cap-weighted index over the risk-free rate. The other six factors are equal-weighted daily rebalanced factors obtained from Scientific Beta. Coefficients significant at 5% p value are highlighted in bold.

2. Performance breakdown



Stylised illustration

important to consider exposures to factors other than the market, one also needs to recognise that the market exposure heavily conditions the performance of multi-factor strategies. The market beta of smart beta strategies is an implicit result of various construction choices but most smart beta offerings have a market beta that is uncontrolled and often lower than one due to the defensive bias of some factors and weighting schemes. This market beta, if left uncontrolled, can lead to significant biases in performance and such biases are often left undocumented. The first order question (of market risk exposure) is ignored while the second order question (of factor intensity) has taken centre stage.

Biases introduced by leaving the market beta of smart beta strategies unmanaged can have two adverse consequences. Firstly, the strategy could be losing out on the long-term equity market risk premium which accounts for a significant portion of long-term performance of any long-only equity strategy. Defensive smart beta strategies with a market beta of less than one lose out on some of this market risk premium (but this performance obviously also comes with lower risk). Secondly, uncontrolled market risk exposure can also produce pronounced differences in short-term performance since market exposure heavily influences the conditional performance of multi-factor strategies. Defensive factor strategies will tend to underperform in bull markets but strongly outperform in bear markets, whereas aggressive factor strategies tend to outperform in bull markets but underperform in bear markets. It is thus crucial to document market beta biases to be able to reconcile them with investor preferences and to correct them, if necessary. Ultimately, the trade-off posed by many multi-factor strategies between a possible reduction in outperformance potential and a strong reduction in volatility, which leads to a clear improvement in the Sharpe ratio over the long term, should be made explicit and should be validated by the stakeholders in the investment.

The importance of market exposure for the performance of smart beta strategies is readily observable from a factor performance attribution analysis of such strategies. As an illustration, figure 1 shows the factor exposures (betas) and performance attribution of the EDHEC-Risk US Long-Term version of Scientific Beta's flagship US High-Factor-Intensity Multi-Beta Multi-Strategy 6-Factor EW index, which is constructed by equally weighting a combination of six single-factor tilted sub-indices. These sub-indices are designed to be exposed to the size, value, momentum, low volatility, high profitability and low investment factors. A diversified weighted scheme is used to construct each sub-index. We regress the returns of the multi-factor strategy on a seven-factor model that includes the market factor and six targeted factors to obtain the factor exposures.

The importance of the market exposure of the strategy is clearly documented in figure 1. The results of this regression show that the multi-factor index indeed has statistical and significant exposure to all six targeted factors and an exposure of 0.97 to the market factor. It is interesting to see that the exposure to the market factor is responsible for 5.71% annualised returns – ie, more than half of the performance of the strategy. The impact of other factors is much smaller. The size exposure contributes 0.34% of the performance, while the momentum, low investment and high profitability factors contribute about 0.17–0.19% of the returns each. Therefore, properly accounting for market exposure in evaluating, selecting and implementing smart beta strategies is an imperative for sound decision-making with respect to such strategies.

In fact, the cost of deviating from a market beta of one can be estimated in a straightforward fashion. In figure 2, we report the cost of being underexposed to the market for the EDHEC-Risk US Long-Term version of Scientific Beta's High-Factor-Intensity Multi-Beta Multi-Strategy 6-Factor EW index. Figure 2 illustrates the different components of performance. The multi-factor strategy generates a positive return in excess of what is explained by its market exposure. This component is what motivates smart beta investors to pursue the strategy, and is driven by better diversification and better factor tilts of the strategy, relative to the cap-weighted market index. However, at the same time, the strategy has lower market exposure than the cap-weighted market index, which leads to a cost of underexposure. This cost is given by the difference in market beta of the multi-factor strategy less a market beta of one, multiplied by the long-term market premium. The multi-factor strategy gives up market exposure (by an amount $\Delta\beta$), which results in giving up market returns. The cost of this underexposure is more than offset by the higher returns generated by the other components of the strategy.

Figure 3 shows the numerical estimates of the different components. The cost of under-exposure is equal to 1.08% (annualised returns) and offsets some of the performance generated on a market risk-adjusted basis (4.58%). In a nutshell, while the strategy manages to generate 4.58% of outperformance over its market risk-adjusted benchmark, it also gives up 1.08% of returns relative to a strategy with a market beta of one due to the under-exposure to the market. Thus, a significant fraction of outperformance potential is lost due to this under-exposure to the market, but this under-exposure to the market also results in a commensurate reduction in the volatility of the strategy.

The market exposure of the strategy also has an even more important

3. Cost of under-exposure to the market

CAPM market beta	0.82
Δ market beta	−0.18
Annualised market factor returns	5.87%
Cost of underexposure to market	**−1.08%**
Returns adjusted for market exposure	4.58%
Average outperformance (arithmetic) over market	3.50%

Universe is EDHEC Risk US Long Term Track Records. Time period of analysis is from 31 December 1975 to 31 December 2015 (40 years). The regressions are based on weekly total returns in US dollars and returns are computed using daily total returns in US dollars. Returns are annualised. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. The market factor is the average excess return series of the cap-weighted index over the risk-free rate. 'Returns adjusted for market exposure' is the abnormal returns of the strategy that is not explained by the market factor in CAPM. 'Average outperformance (arithmetic) over market' is the difference between the average strategy excess returns (over risk-free) and average cap-weighted excess returns (over risk-free). It is different from 'relative returns' over CW, which is simply the difference of average strategy returns and average cap-weighted returns.

4. Impact of market beta on risk

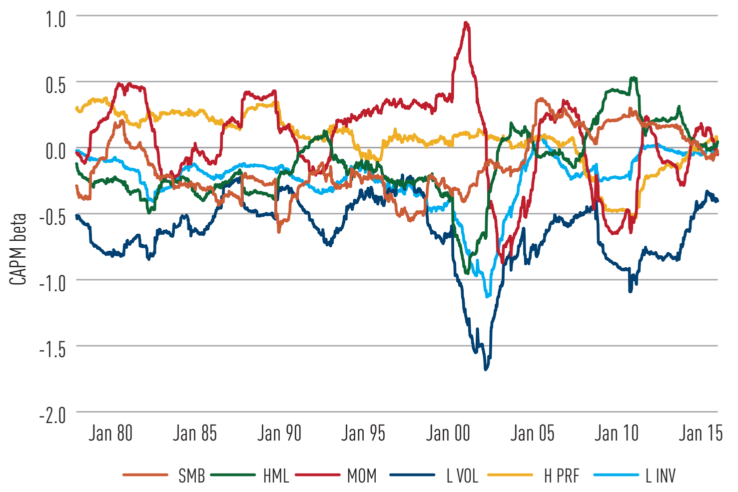
Factor exposure	
Annualised unexplained	**1.92%**
Market beta	**0.97**
SMB beta	**0.18**
HML beta	**0.14**
MOM beta	**0.04**
Low volatility beta	**0.14**
High profitability beta	**0.09**
Low investment beta	**0.07**

Volatility attribution	
Idiosyncratic component	0.79%
Market factor	**17.55%**
SMB factor	0.23%
HML factor	0.20%
MOM factor	0.03%
Low volatility factor	0.04%
High profitability factor	0.05%
Low investment factor	0.04%
Interaction component	–5.53%

Universe is EDHEC Risk US Long Term Track Records. Time period of analysis is from 31 December 1975 to 31 December 2015 (40 years). The analysis is based on weekly total returns in US dollars. All statistics are annualised. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. The market factor is the excess return series of the cap-weighted index over the risk-free rate. The other six factors are equal-weighted daily rebalanced factors obtained from Scientific Beta. Coefficients significant at 5% p value are highlighted in bold.

5. Market beta of factors

Dollar-neutral L/S factors US long-term – rolling CAPM beta (2y-1w)



Universe is EDHEC Risk US Long-Term Track Records. Time period of analysis is from 31 December 1975 to 31 December 2015 (40 years). The analysis is based on weekly total returns in US dollars. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate. The market factor is the excess return series of the cap-weighted index over the risk-free rate. Rolling window of length two years and step size one week is used.

impact on the return variability of the multi-factor index. Figure 4 shows a risk attribution analysis of the multi-factor strategy. The volatility contribution of the market factor is 17.55%, while each of the other six factors contributes less than 0.5%.

Not only is the market factor a strong contributor to the overall risk of a long-only multi-factor allocation but this market beta is also highly time varying. Figure 5 shows two-year rolling market betas (from a CAPM model) of US long-term dollar-neutral long/short factors from Scientific Beta. Dollar-neutral long/short factor portfolios are typically used to assess factors since the market exposure of the long-leg is mitigated by the market exposure of the short leg. Figure 5 shows that the factors' market betas are highly variable and this variability is expressed strongly even in a long/short framework.

Some factors like low volatility and low investment are associated with negative CAPM beta but the variation in the magnitude of beta is quite high. The other four factors show both positive and negative CAPM beta depending on the period, with momentum showing rather cyclical behaviour. During bull periods, the momentum factor loads on high-market-beta stocks and during falls in the market, it is exposed to low-market-beta stocks as these stocks are the least badly hit in terms of performance. Daniel and Moskowitz (2016) also show that momentum-tilted portfolios tend to rebalance to low (high) beta stocks following periods of low (high) market returns.

The impacts of this variation in beta on the conditionality of performance are considerable, especially since the factors' market betas can have poor conditional qualities. The negative consequences of highly conditional market betas of factors can be corrected by making the factors beta-neutral. The CAPM beta over a 40-year period of the long and short legs of dollar-neutral long/short (L/S) factors is used to leverage/de-leverage the long and short legs to beta 1.

Figure 6 compares the extreme conditional performance of the six US long-term long/short factors in dollar-neutral form and in a form that controls for the market beta (statically beta-neutral form). The low volatility and low investment factors, which have negative market exposure in dollar-neutral form, exhibit poor performance in extreme bull markets and good performance in extreme bear markets. This is as a direct consequence of their negative exposure to the market factor. The statically beta-neutral factors correct this conditional performance asymmetry.

From an investor's perspective, the dependency of the (uncontrolled) factors on market conditions leads to an implicit exposure that may not be desirable. In the case of the value factor for example, we observe that the uncontrolled factor generates profits in bear markets while generating losses in bull markets. However, the motivation for gaining value exposure is definitely not to make implicit bets on the market but rather to harvest the value premium. Therefore, a relevant question is how to implement factor exposures while controlling market exposure.

Given the importance of the market risk factor it seems surprising that – up until now – the market risk for the vast majority of multi-factor offerings had not been managed. In fact, while there have been heated debates about which precise definition to use for non-market factors (see for example Blitz et al [2011] or Rao et al [2015] among many others), whether or not such factors are overpriced (see the debate in Arnott et al [2016] and Asness [2016]), and how to control the intensity of exposure to such non-market factors (see Bender and Wang [2016] or Clarke et al [2016] among many others), there has been little, if any, discussion on controlling market risk. Indeed, one might argue that, in focusing on the second order question (of the exposure to factors other than the market), smart beta providers have neglected the first order question (exposure to the market factor).

6. Market beta bias in conditional performance

1975–2015	*Dollar-neutral* L/S factors US Long Term					
	SMB	HML	MOM	L VOL	H PRF	L INV
Extreme bull returns	8%	–3%	2%	–22%	5%	–8%
Extreme bear returns	–7%	9%	6%	53%	1%	35%

1975–2015	Statically *beta-neutral* L/S factors US Long Term					
	SMB	HML	MOM	L VOL	H PRF	L INV
Extreme bull returns	13%	1%	3%	6%	4%	3%
Extreme bear returns	–11%	4%	5%	12%	1%	17%

Universe is EDHEC Risk US Long Term Track Records. Time period of analysis is from 31 December 1975 to 31 December 2015 (40 years). The analysis is based on weekly total returns in US dollars. All statistics are annualised. Bull market is composed of quarters that have positive market returns. Bear market is composed of quarters that have negative market returns. Extreme bull quarters are top 50% of bull quarters with the best market returns. Extreme bear quarters are bottom 50% of bear quarters with the worst market returns. The CAPM beta over 40-year period of the long and short legs of dollar-neutral L/S factors is used to leverage/de-leverage the long and short legs to beta 1. The statically beta-neutral L/S factors are obtained as the excess return of unit beta long leg over unit beta short leg.

Hidden macro sensitivities

Factor strategies are also exposed to macroeconomic risks, something investors may not be aware of, as they are not documented by most providers. Biases in macro sensitivities will also impact performance similar to market risk bias. Factor sensitivities to macroeconomic risks lead to different performance in different macro conditions.

More importantly, macro exposure biases will lead to interaction effects with other factors and other asset classes. For example, strategies with sensitivity to credit risk or interest rate risk will interact with fixed-income portfolios. A multi-factor portfolio may lack diversification across factors if several risk factors are sensitive to the same macro driver. In this case having a constant and balanced (beta) exposure to multiple rewarded risk factors may not necessarily improve the diversification of the portfolio. Focusing instead on the diversification of the factor risk premia within a macroeconomic regime is more important that maintaining a balanced exposure across multiple factors. In this section, we illustrate the macro sensitivities of equity factors to a set of relevant macro-variables.

We will assess the conditional performance of six well-known long/short equity risk factors conditioned upon various macroeconomic variables. Calendar months are sorted into quartiles according to each conditioning variable, and monthly average returns are compared. For the sake of brevity, we only report spreads between extreme quartiles. For example, in the case of inflation, the reported figure will correspond to the difference in average monthly returns between 25% of calendar months when inflation was highest versus lowest. We use EDHEC-Risk Long-Term Track Records over the past 40 years. The macroeconomic variables that the returns of the factors are conditioned on can be broadly grouped into four categories and are shown in figure 7.

Figure 8 shows the conditional performance of the six long/short factors using the EDHEC-Risk US Long-Term Track Records (31 December 1975 to 31 December 2015).

Some factors reveal opposite sensitivity to a number of macroeconomic variables, suggesting there is room for offsetting sensitivity to macroeconomic variables by suitably designed factor combinations. For example, size and low volatility have complimentary exposure to several macroeconomic variables and value and momentum have strong complimentary sensitivity to changes to term spread.

The sensitivity to macro variables is most pronounced for the low volatility factor and the size factor was also influenced by a large number of conditioning variables. Value and high profitability were the least dependent factors to the selected variables. Most factors were sensitive to sector spread and change in dividend yield, while industrial production, inflation and liquidity had no influence on factor premia.

Part of the sensitivity of factors to macroeconomic variables may be due to non-zero market exposures of the long/short factors. If the market is sensitive to the conditioning variable, and if the given factor is exposed to market risk, analysis may confound market exposure and macro exposure. To eliminate the market effect, in figure 9 we look at the CAPM alpha instead of absolute returns.

Figure 9 shows the low volatility and low investment factors are no longer sensitive to market returns and dividend yield after adjusting for market exposure, but still negatively exposed to the sector spread. The size, value and momentum factors respond in a similar way to the different macro variables after controlling for market beta. Again, the high profitability factor appears as the least responsive factor to macroeconomic variables. Overall, we see that the difference between factor returns in different regimes is partly driven by market exposure.

This strong difference in the macroeconomic conditionality of market-beta-adjusted versus non-market-beta-adjusted factor strategies has important consequences, notably when it involves qualifying factor regime premia and trying to predict the future returns of the factors associated with these premia. Many long-only active managers who are opposed to the difficulty (impossibility) of producing alpha in traditional tactical allocation are just repackaging doubtful market return forecasting skills as new factor timing skills! This difficulty stems from the complexity in predicting future market returns with the expected robustness of factor regime premia forecasting (forgetting that a large share of the observed conditionality of factor returns is in fact related to market beta).

7. Macroeconomic variables used in conditional analysis

Group	Variable	Definition
Stock market indicators	Market returns	Returns on Scientific Beta cap-weighted market index. Source: Scientific Beta
	Change in market volatility	Standard deviation of daily returns on Scientific Beta cap-weighted market index computed monthly. Source: Scientific Beta.
Economic indicators	Industrial production growth	The Industrial Production Index (INDPRO), seasonally adjusted. Source: Board of Governors of the Federal Reserve System (US).
	Sector spread	Defined as a difference between returns on cyclical and defensive sectors, according to MSCI classification.
	Inflation	Returns on a seasonally adjusted Consumer Price Index have been used as a proxy for inflation. Source: US Bureau of Labour Statistics.
	Change in aggregate traded volume	Defined as a ratio of total daily dollar volume traded in the universe over aggregate market capitalisation of the universe in US dollars.
Asset pricing indicators	Change in term spread	Defined as a difference between yields on 10-year and 1-year government bonds.
	Change in default spread	Defined as spread between Moody's AAA and BAA corporate bonds.
	Change in 12-month dividend yield	Defined as the difference between log returns of total return index and price index of broad cap-weighted index.
Other asset classes	Sovereign bond returns	Returns on Barclays US Treasury Bond Index have been used as a proxy for sovereign bond returns.
	Commodity returns	S&P GSCI index. Source: S&P Dow Jones Indices.
	Currency returns	Broad trade-weighted US dollar index is used as proxy. Source: Board of Governors of the Federal Reserve System (US).

8. Conditional performance – long-term United States

EDHEC-Risk US LTTR 31 Dec 1975–31 Dec 2015	Size	Value	Momentum	Low volatility	High profitability	Low investment
Market returns	0.58	–0.53	0.15	**–8.02**	–0.32	**–3.09**
Change in market volatility	–2.03	0.01	–0.33	**2.40**	0.07	0.53
Sector spread	**2.96**	–0.06	–1.27	**–8.03**	0.29	**–2.93**
Industrial production growth	–0.47	0.71	–0.08	–0.21	–0.56	0.21
Inflation	0.24	–0.03	0.62	0.48	0.21	0.07
Change in term spread	0.21	**1.15**	**–2.12**	–1.05	–0.30	0.08
Change in default spread	**–1.04**	0.08	–0.63	1.04	–0.09	0.17
Change in dividend yield	**–1.10**	–0.73	0.88	**6.57**	**1.10**	**1.89**
Sovereign bond returns	**–1.00**	–0.81	0.94	**2.22**	–0.43	0.16
Commodity returns	**1.57**	–0.05	**1.65**	**–2.76**	–0.07	–0.02
Currency returns	**–0.41**	0.14	0.25	**1.63**	0.00	–0.06
Change in average traded volume	–0.82	–0.77	–0.02	0.09	0.65	–0.47

The analysis is based on daily total returns in US dollars from 31 December 1975 to 31 December 2015. The market factor is the excess returns of cap-weighted benchmark over risk free rate. The other factors are long/short portfolios with long leg comprising of top 30% stocks based on the corresponding factor score and short leg comprising of bottom 30% stocks based on the corresponding factor score. The reported figures correspond to the return spreads between extreme quartiles according to the conditioning variable (Q4–Q1). The numbers that are statistically significant at 5% are reported in bold. A pale blue fill indicates that average returns from the bottom to top quartiles are increasing/decreasing monotonously. The most influential variable across each factor is highlighted in red.

9. Conditional CAPM alphas – long-term United States

EDHEC-Risk US LTTR 31 Dec 1975–31 Dec 2015	Size	Value	Momentum	Low volatility	High profitability	Low investment
Market returns	0.25	–1.47	1.57	0.51	0.47	–1.21
Change in market volatility	**–1.90**	0.12	–0.58	0.19	–0.23	–0.21
Sector spread	**3.14**	–0.08	–1.01	**–4.80**	0.72	**–1.84**
Industrial production growth	–0.47	0.78	–0.34	–0.65	–0.60	0.02
Inflation	0.44	0.04	0.41	–0.56	0.04	–0.28
Change in term spread	0.13	**1.09**	**–1.98**	–0.73	–0.21	0.21
Change in default spread	**–1.02**	–0.06	–0.58	0.62	–0.11	0.06
Change in dividend yield	–1.37	–0.83	–0.12	–1.02	**1.10**	–0.85
Sovereign bond returns	**–1.04**	–0.80	0.91	**2.54**	–0.39	0.30
Commodity returns	**1.46**	–0.14	**1.87**	**–1.42**	0.11	0.46
Currency returns	–0.25	0.19	0.15	0.65	–0.04	–0.39
Change in average traded volume	–0.68	–0.70	–0.29	–0.96	0.56	**–0.90**

The analysis is based on daily total returns in US dollars from 31 December 1975 to 31 December 2015. The market factor is the excess returns of cap-weighted benchmark over risk free rate. The other factors are long/short portfolios with long leg comprising of top 30% stocks based on the corresponding factor score and short leg comprising of bottom 30% stocks based on the corresponding factor score. The reported figures correspond to the return spreads between extreme quartiles according to the conditioning variable (Q4–Q1). The numbers that are statistically significant at 5% are reported in bold. A pale blue fill indicates that average returns from the bottom to top quartiles are increasing/decreasing monotonously. The most influential variable across each factor is highlighted in red.

Exposure to sector and country-specific risks

Managing unrewarded risks is one of the key challenges faced by smart beta investors. Index providers are not always transparent about the implicit unrewarded bets their offerings are exposed to. Sector and region effects are important in explaining the variation in stock returns per se, but these are unrewarded risks over the long term and therefore have to be validated by investors, because there is no empirical or academic evidence for taking these risks.

Figure 10 shows smart beta strategies can lead to strong biases compared to the cap-weighted benchmark, if left unmanaged. These biases often have more impact on performance in the short and medium term than the risk premium associated with low volatility over the long term.

In addition, although investors are often encouraged to look at the factor intensity alone as an explanation for good back-tested performance, it should be stressed that the way in which one obtains this factor intensity is not neutral from the perspective of exposure to unrewarded risk. In our view, it is a shame not to document this subject from the viewpoint of the fiduciary responsibility of the investor or their asset manager. Figure 11 shows that bottom-up approaches to multi-factor portfolio construction may be influenced substantially by sector biases.

Naturally, documenting does not necessarily mean neutralising. For example, there is no orthogonality between factor tilts and sector tilts, which are both microeconomic bets. A sector-neutrality constraint necessarily reduces the factor intensity, which reduces absolute risk-adjusted return over the long term but comes with better control of tracking error, which may be desirable for investors whose performance is measured relative to a cap-weighted benchmark and are more concerned about relative risk-adjusted return. On the other hand, if the objective of the factor strategy is to achieve high absolute risk-adjusted returns or if the objective of the investor is to benefit from diversification across factor premia with opposing sensitivity to a sector spread it will not be a good idea to limit this cyclicality by imposing sector constraints. For these reasons, ERI Scientific Beta, which positions itself as an index provider that leaves the choice of fiduciary options to those who have real responsibility for them, offers its multi-factor indices with and without sector-neutrality as a risk control option depending on the requirements of the investor. Comparing the performance of such indices with and without sector control allows documenting the importance of sector risks.

In the same way, smart beta strategies correspond too frequently to optimisations at the stock level that do not take the geographical risk into account. This risk is not 'naturally' rewarded and should therefore be subject to an explicit decision. Figure 12 shows that smart beta strategies can lead to strong regional biases compared to the cap-weighted benchmark.

Unlike sector risk, there is no real and serious trade-off here between taking justified macroeconomic and microeconomic risks. Taking macroeconomic risks on the basis of academic reasoning and proofs that correspond to a pure microeconomic dimension makes no sense and is ultimately disrespectful to the stakeholders in the investment. ERI Scientific Beta suggests neutralising these risks at the regional level consistently on the basis of respecting the relative weight in market capitalisation of each region. This regional approach reconciles smart beta and factor investing and controls unrewarded geographical risks. Factor investing has been documented to work best when performed within economically-integrated regions. Following the rejection of the global model by Griffin (2002), Fama-French (2012) build regional models that support index construction at the block-level. There is no justification for using microeconomic factors to take macroeconomic bets. ERI Scientific Beta offers its single and multi-factor strategies also on a regional basis allowing an investor to take active regional bets.

10. Exposure to sector biases

Developed 16 December 2016	Allocation MSCI World	 MSCI World Min Vol	Relative allocation MSCI World Min Vol
Energy	7.3%	2.1%	–5.2%
Basic materials	5.0%	2.6%	–2.3%
Industrials	11.2%	8.7%	–2.6%
Cyclical consumer	12.3%	8.4%	–3.9%
Non-cyclical consumer	9.6%	14.8%	5.2%
Financials	21.1%	19.6%	–1.5%
Healthcare	11.9%	16.9%	5.0%
Technology	14.7%	10.0%	–4.7%
Telecoms	3.3%	8.4%	5.1%
Utilities	3.1%	8.2%	5.1%
Others	0.5%	0.4%	–0.1%
Sum of absolute weight deviations with respect to MSCI World			**40.70%**

11. Bottom-up multi-factor portfolios and sector bias

EDHEC RIsk US LTTR 31 Dec 1975–31 Dec 2015	Cap-weighted	Score times cap-weighted MFS 20% stock selection based on Arithmetic average	 Geometric average	 MFS 50% stock selection based on Arithmetic average	 Geometric average
Effective number of sectors	7.95	6.18	6.53	7.85	7.62
Drift of effective number of sectors	3.78%	11.76%	9.68%	8.31%	4.94%
Drift of sector weights	4.39%	19.77%	14.30%	12.28%	8.68%
Sector bias (sum of absolute difference in weights)	–	68.31%	66.00%	42.65%	51.53%
Drift of sector bias	–	13.62%	12.69%	7.08%	7.31%
Drift of active sector weights	–	18.94%	14.09%	10.90%	7.92%

Based on time period between 31 December 2005 and 31 December 2015. The effective number of sectors is the inverse of the sum of squared weights allocated to each sector, and is averaged across all rebalancing dates. The drift of effective number of sectors is standard deviation of effective number of sectors divided by effective number of sectors. The drift of sector weights is the square root of the sum of sector weights' variances. The sector bias is the sum of absolute active sector weights, which is the differences between the sector weights of cap-weighted benchmark and respective index, and is averaged across all rebalancing dates. The drift of sector bias is the standard deviation of sector bias. The drift of active sector weights is the same as drift of sector weights using active weights of sectors instead of absolute weights. The sectors are classified according to Thompson Reuters Business Classification. The shorter time-period of analysis is due to limited data availability.

Finally, one should note that unlike in developed world regions, countries within emerging market regions are not as highly integrated and figure 13 shows that factor indices may display large deviations in country exposures compared to the cap-weighted benchmark.

Figure 14 shows these large deviations in country exposures lead to high relative risk and low information ratios. Controlling for country risks (geo-neutral) reduces tracking error and maximum relative drawdown and leads to an improvement in the information ratio compared to the non-country-neutral version. Here a very clear decision needs to be taken between the search for a better Sharpe ratio over the long term, which authorises the

12. Exposure to regional biases

Developed 16 December 2016	Allocation MSCI World	 MSCI World Min Vol	Relative allocation MSCI World Min Vol
US	60.6%	63.2%	2.6%
Japan	8.7%	13.8%	5.1%
UK	6.6%	2.8%	–3.8%
Canada	3.6%	1.1%	–2.6%
France	3.6%	0.1%	–3.5%
Germany	3.3%	0.3%	–3.0%
Switzerland	3.1%	5.2%	2.2%
Australia	2.6%	0.3%	–2.3%
Hong Kong	1.2%	3.8%	2.6%
Other	6.7%	9.5%	2.8%
Sum of absolute weight deviations with respect to MSCI World			**30.50%**

13. Country allocation – emerging markets

Emerging markets diversified multi-strategy indices (excess weights) Low volatility	Standard	Geo-neutral	Value	Standard	Geo-neutral
Top 3 countries by excess weights					
Malaysia	5.73%	–0.03%	Taiwan	4.98%	–0.15%
Taiwan	5.24%	1.41%	Malaysia	2.67%	0.13%
Chile	3.83%	0.16%	Thailand	2.26%	0.64%
Bottom 3 countries by excess weights					
China	–10.63%	–0.11%	India	–6.11%	2.72%
Brazil	–6.23%	–0.23%	South Africa	–4.36%	0.67%
South Korea	–4.88%	–1.64%	China	–2.16%	–2.26%

Country allocation: Analysis is based on SciBeta emerging indices data as of 17 March 2017. SciBeta Emerging Broad cap-weighted index is used as the benchmark. Country wise excess weights with respect to cap-weighted benchmark is shown here. The top and bottom 3 countries based on excess weights of standard indices are included here.

14. Performance – emerging markets

Emerging markets 31 Dec 2006–31 Dec 2016	Broad cap-weighted	Emerging markets diversified multi-strategy indices Low volatility		Value	
		Standard	Geo-neutral	Standard	Geo-neutral
Annualised return	2.51%	6.88%	6.07%	4.88%	4.78%
Volatility	21.16%	15.28%	17.44%	18.09%	19.34%
Sharpe ratio	0.06	0.36	0.27	0.20	0.18
Maximum drawdown	64.28%	50.74%	56.28%	56.32%	61.06%
Relative returns	–	4.38%	3.56%	2.37%	2.28%
Tracking error	–	8.08%	4.96%	5.58%	4.13%
95% tracking error	–	17.20%	9.78%	12.58%	8.13%
Information ratio	–	0.54	0.72	0.43	0.55
Maximum relative drawdown	–	18.46%	9.62%	10.04%	6.59%

Long-term performance: Analysis is based on SciBeta emerging indices data and daily total returns in US dollars are used. The time period is 31 December 2006 to 31 December 2016. SciBeta Emerging Broad cap-weighted index is used as the benchmark. Three-month US Treasury Bill rate is used as proxy for risk-free rate.

control of factor investing at the country level, versus the better management of conditionality and relative risk with respect to reference cap-weighted indices that the country-neutral option allows.

Conclusion

Smart beta investors are subject to several risks that most providers fail to report on reliably. These undocumented risks can have a significant impact on performance. Documenting such risk exposures is crucial to reconcile them with investors' preferences. With cap-weighted indices, which represent the default option in terms of a passive investment reference, being increasingly called into question, smart beta's main fiduciary message is that there is no best solution in general, but rather a best solution that allows the investor's fiduciary choices to be executed in the most efficient way. This is probably what best defines the difference between passive investment and active investment at a time when the former is no longer static and brings its own promise of outperformance compared to the cap-weighted index.

Future challenges for smart beta index providers are to address fully the implications of smart beta risk exposures. An industry-wide effort is needed to improve disclosure of risks. Every risk is an opportunity (for risk management). Documenting all the risks is a necessary condition for managing those risks through new innovative solutions. Ultimately, the choice on managing these risks is a key fiduciary decision that cannot be left to the appreciation of an index provider who has no status to do so.

Asset owners' governance practices should also be improved by starting a risk conversation on smart beta investments with stakeholders. Which risks are desired and undesired? How to align risks with investment beliefs? How to evaluate and manage interaction across the policy portfolio?

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Assessing the investability of smart beta indices

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It is obviously crucial to assess the investability of smart beta strategies, as they naturally incur additional implementation hurdles compared to cap-weighted indices.

However, current performance reporting practices in the market lack sufficient information regarding the investability of smart beta indices.

While there are different dimensions related to investability, such as liquidity, capacity and transaction costs, it is possible to provide transparency on these dimensions with a range of metrics developed in market microstructure research.

This article introduces a suite of analytics to enable investors to assess the investability of smart beta indices.

With the advent of smart beta equity indices, which represent alternatives to market cap-weighted indices, a major question on their investability has been raised: at what cost will investors be able to trade the index constituents in the same proportions as the underlying strategy?

In fact, departing from the traditional cap-weighting investment scheme leads to risks that are sizable and significantly different, as shown in Amenc, Goltz and Lodh (2012) and Amenc, Goltz and Martellini (2013). These include common exposures to systematic risk factors such as size and liquidity.

Also, in contrast to cap-weighted indices, which are deemed to be buy-and-hold investments, and which are only marginally reviewed for the (often quarterly) addition and deletion of constituents as well as regular corporate events, smart beta indices exhibit higher levels of turnover than their cap-weighted counterparts (see, eg, Amenc et al [2011]).

Importantly, for any level of liquidity, the level of turnover in the index will impact the performance of the tracking fund through the frequency of occurrence of transaction costs. Clearly, investing in smart beta indices requires investors to have access to solutions where implementation costs and liquidity risks are thoroughly considered.

Providers and investors agree that smart beta strategies incur additional costs to trade compared to cap-weighted indices; however, what remains unanswered is how to measure these costs reliably. Typical backtest performances of smart beta strategies offered by commercial index providers do not consider real-life transaction costs. Index providers prefer to leave it to market participants to figure out what the costs of trading those strategies would be or they rely on some arbitrary assumptions on transaction cost levels. At the same time, there are also some market participants who make bold claims that smart beta strategies experience a significant drag in terms of implementation costs, to the extent of rejecting the value-add of all smart beta strategies. Unfortunately, such claims are not usually accompanied by actual measures of implementation costs. In the absence of actual costs, such claims cannot be evaluated.

In this article, we address this gap by introducing a suite of analytics developed by ERI Scientific Beta specifically to enable investors to assess the investability of smart beta indices and help them make informed decisions on whether smart beta indices add value in excess of the costs incurred to implement them. We briefly describe how the metrics are defined and their utility in assessing the investability of the indices along with a performance report of their application to actual indices.

Investability metrics

ERI Scientific Beta has developed a set of analytics specifically to assess the investability of our indices, given that there is a significant gap in that area in performance reporting in the industry. For example, there is extensive academic literature that discusses simple ways to assess the trading costs of smart beta strategies, yet index providers do not seem to apply them in assessing their actual cost estimates. Price impact is another example of a hidden cost that is often ignored. In order to address these shortcomings in performance reporting regarding the investability of smart beta indices, ERI Scientific Beta has developed a set of analytics that provides a comprehensive analysis of the investability of its indices. In this section, we describe these investment analytics briefly, especially their definition, economic interpretation and finally the empirical assessment of the investability of our smart factor indices using these analytics. The investability of smart beta indices can be assessed according to different dimensions such as liquidity, capacity, transaction costs, etc. ERI Scientific Beta's investability analytics comprehensively cover these different dimensions. We will thoroughly assess the investability of Scientific Beta's smart factor indices along these various dimensions using these analytics.

The investability analytics are categorised into three groups.

- Turnover and transaction costs;
- Liquidity indicators; and
- Days-to-trade and price impact.

Below we review each of these in detail.

Turnover and transaction costs

Turnover

Turnover refers to the measurement of how frequently, and in which relative proportions, the constituents of an equity strategy index are traded over a specific period. Turnover, which leads to transaction costs that are higher than those of buy-and-hold strategies and which may make it harder to replicate the index, is of concern to index investors.

The turnover is calculated as the sum of absolute deviations of individual weights (or positions) between the end of a quarter and the beginning of the following quarter.

Transaction costs

Transaction costs play a major role in the performance drag of smart beta strategies. In a recent EDHEC-Risk Institute study (Esakia et al [2017]) the authors compute intra-day spread estimates using low frequency data (daily data) by drawing upon recent advances in market microstructure literature and applying them to popular smart beta strategies. We draw upon that study to compute transaction costs for all of our smart factor offerings. Total trading costs can be decomposed into three components: the spread, the price impact, and commissions (see, eg, Hasbrouck [2007]). Commissions are usually insignificant for large institutional investors and are very institution-specific. The spread reflects the cost of a round-trip trade (buy and sell). The basic cost definition is the percentage quoted spread which reflects the percentage cost of buying at the ask quote and selling at the bid quote. If trades occur at prices different from the best bid and offer (BBO), the percentage effective spread is a more useful measure because in the case of large orders the effective spread captures the price impact caused by the trade as well as the realised spread (Huang and Stoll [1996]).

Estimating effective spread requires high-frequency intra-day data. We need to observe trades and quotes within the trading day to come up with cost measures. However, such data is both hard to use and hard to get. It is hard to use because high-frequency data is big and messy. The increasing frequency of trading has led to a huge amount of tick-by-tick price data requiring massive computational power for analysis. Fong, Holden and Trzcinka (2014), who analyse several billion data points, argue that high-frequency equity data likely grows at a rate of more than 30% per year, which outpaces the growth of computing power. Moreover, tick data requires price and quote procedures to be matched (see Lee and Ready [1991]), and intense data cleaning, so that the quality of databases and the cleaning procedures become a prime concern. High-frequency data is hard to get because it is expensive, and it covers only short time periods. It is common for researchers to analyse only periods of less than a decade, sometimes only a few years, due to data availability limitations.

Fortunately, recent research has shown that there are effective ways of estimating transaction cost variables that are only observable at high frequency based on lower frequency (daily) data. The advantage of such approaches is that results can be generated for longer periods and different markets, with relative computational ease and limited data needs. There are several low-frequency measures proposed in the literature that are efficient proxies for the effective spread. We chose the method introduced by Chung and Zhang (2014) as it is widely considered the best proxy for the intra-day effective spread (see Fong, Holden and Trzcinka [2014], Chung and Zhang [2014] and Abdi and Ranaldo [2017]). Chung and Zhang (2014) show that the daily closing spread is a good proxy for the intra-day effective spread, which includes both a realised spread component and a price impact component. We use the same methodology and use the closing spread to compute the transaction costs for implementing a strategy by multiplying one-way turnover of a portfolio by the average closing bid-ask spread in the given universe.

Empirical results – turnover and transaction costs

Scientific Beta smart factor indices have a manageable turnover because of the turnover control employed. They all are in line with the ex-ante turnover targeted by the turnover control rules. Overall, the average relative returns of all the indices are much higher compared to the transaction costs, thus showing that increased costs in adopting Scientific Beta indices over cap-weighted benchmarks are well compensated by means of increased returns.

1. Turnover and transaction costs

31 December 2006–31 December 2016

	Broad cap-weighted	High Factor Intensity Diversified Multi-Strategy indices						
		Mid cap	Value	High momentum	Low volatiity	High profitability	Low investment	Multi-Beta Multi-Strategy (6 Factor) EW
US								
Annualised one-way turnover	4.11%	45.13%	37.35%	80.45%	32.58%	30.13%	43.39%	38.30%
Transaction costs	< 0.01%	0.03%	0.02%	0.05%	0.02%	0.02%	0.03%	0.02%
Relative returns	–	2.28%	2.43%	1.21%	3.29%	2.86%	2.38%	2.44%
Relative returns net of costs	–	2.25%	2.41%	1.17%	3.27%	2.84%	2.36%	2.41%
Developed world								
Annualised one-way turnover	4.15%	46.32%	38.33%	79.62%	35.48%	32.61%	45.89%	39.70%
Transaction costs	0.01%	0.12%	0.10%	0.21%	0.09%	0.09%	0.12%	0.11%
Relative returns	–	2.84%	2.56%	2.24%	3.52%	3.78%	3.23%	3.04%
Relative returns net of costs	–	2.72%	2.46%	2.03%	3.43%	3.69%	3.10%	2.94%
Emerging markets								
Annualised one-way turnover	9.38%	54.82%	39.95%	77.43%	42.86%	35.85%	48.14%	43.57%
Transaction costs	0.04%	0.26%	0.19%	0.37%	0.20%	0.17%	0.23%	0.21%
Relative returns	–	5.79%	4.25%	3.07%	5.34%	4.81%	5.59%	4.84%
Relative returns net of costs	–	5.53%	4.06%	2.70%	5.14%	4.64%	5.36%	4.63%

Analytics are calculated over the period 31 December 2006 to 31 December 2016. Transaction costs is the product of average spread in the respective universes (closing spread proposed in K. H. Chung and H. Zhang (2014). A Simple Approximation of Intraday Spreads with Daily Data. *Journal of Financial Markets* 17: 94–120) and one-way turnover of the index. Source: www.scientificbeta.com.

Liquidity indicators

Market capitalisation (average, cumulative)
The market capitalisation of an index provides a measure of the overall size of the constituents in the index. The indices that use alternative weighting schemes (ie, non-market-cap-weighted indices) have a natural small-cap bias. The magnitude of this bias can be analysed by comparing the market capitalisation of the alternative index with that of its cap-weighted benchmark.

Scientific Beta reports two measures of market capitalisation: average and cumulative market capitalisation. The average market capitalisation is the weighted average of the free-float market capitalisation of the stocks in the index, whereas the cumulative market capitalisation is the number of stocks in the index times the weighted average free-float market capitalisation of the stocks in the index.

Absolute and relative volume (average, cumulative)
The volume of trading associated with a stock can be used as a measure of its liquidity. Scientific Beta reports two measures of absolute volume: average and cumulative absolute volume. The average absolute volume is the weighted average of the average daily traded volume (ADTV) of the stocks in the index; whereas the cumulative absolute volume is the number of stocks in the index times the weighted average absolute volume of the stocks in the index.

Similarly, Scientific Beta reports two measures of relative volume: average and cumulative relative volume. The relative volume of a stock is measured as its ADTV divided by free-float market capitalisation. The average relative volume is the weighted average of the relative volume of the stocks in the index, whereas the cumulative relative volume is the number of stocks in the index times the weighted average relative volume of the stocks in the index.

Empirical results – liquidity indicators
The single and multi-factor indices have less liquidity than the broad cap-weighted index as one would expect. Except for mid-cap indices, every other factor index has reasonably high liquidity. The mid-cap indices have the lowest average market capitalisation, while the high profitability indices have the highest average market capitalisation.

A similar pattern is observed with average volume traded. The mid-cap indices have the lowest average absolute volume and high profitability indices have the highest average absolute volume. Overall, the indices offer sufficient liquidity for large-scale investments in terms of market capitalisation and trading volume.

Days-to-trade and price impact

Days-to-trade and the Amihud illiquidity ratio both measure the tradability of smart beta indices. Days-to-trade is a proxy that helps to assess the time it takes to trade an index and the Amihud illiquidity ratio is a proxy for the price impact caused by large trades in the index.

Maximum days-to-trade
The maximum days-to-trade of an index reflects the number of days needed to set up an initial investment in the index. The days-to-trade required to set up an initial investment are obviously much higher than those needed for periodic rebalancing. In fact, while periodic rebalancing is restricted by turnover control, the initial investment, by construction, requires 100% of the portfolio to be traded. As each stock in an index is bought to set up the initial investment, the days-to-trade of the portfolio will depend on the maximum value of the days-to-trade of all the stocks in the portfolio. To avoid the reporting of days-to-trade for an index being skewed by an extreme value, one can consider reporting a high percentile value of the cross-section of days-to-trade rather than reporting a maximum of the cross-section of days-to-trade values. In using the volume of a stock to compute days-to-trade, one should also consider that the whole volume of a stock is not available to a single investor, and thus an approximation is made about the percentage of average daily traded volume that is available for trading for an investor. We assume an initial investment of $3bn as of June 2016 with a 10% availability of traded volume and report the 95th percentile days-to-trade value.

Effective days-to-trade
This measure indicates the liquidity stress caused by periodic rebalancing of the index. The effective days-to-trade of an index reflects the number of days needed to trade the changes in positions in an index resulting from index rebalancing. The 'effective days to trade' of a stock is defined as the ratio of the product of a notional investment amount and change in weights due to rebalancing to the average daily trading volume. The assumptions on initial investment and percentage of traded volume available for a single investor are the same as that of the maximum days-to-trade.

Price impact proxy – Amihud illiquidity ratio
One of the biggest concerns with the wide adoption of factor investing by large institutional investors is the expected performance drag induced by large trade orders during the systematic rebalancing of the indices. As any index is systematically rebalanced, the funds tracking the indices adjust their holdings all at the same time, potentially creating a large volume of trades resulting in adverse price movements of the stocks and thereby negatively impacting the index performance. Market impact costs are not easily observed; nevertheless, it is essential to analyse the potential impact of any smart beta strategies. However, index providers provide little information regarding the same for their respective offerings. They leave it to the fund managers to assess the market impact. Amihud (2002) introduced an illiquidity measure using daily stock returns and trading volume as opposed to high frequency trade data. This is one of the most widely used liquidity proxies in the finance literature to measure the price impact of trading. The Amihud measure of a stock is defined as the ratio of the absolute daily return of the stock to the daily dollar volume traded. It measures the change in returns for every dollar traded and is thus widely used as a proxy for price impact.

Empirical results – days-to-trade and price impact
The liquidity constraints applied by Scientific Beta during the index design help in keeping the days-to-trade at a reasonable level. In-line with the other investability analytics,

2. Liquidity indicators

31 December 2006–31 December 2016

	Broad cap-weighted	High Factor Intensity Diversified Multi-Strategy indices						
		Mid cap	Value	High momentum	Low volatiity	High profitability	Low investment	Multi-Beta Multi-Strategy (6 Factor) EW
US								
Average market capitalisation ($m)	98,126	8,888	29,385	32,322	37,986	39,049	30,442	29,685
Cumulative market capitalisation ($m)	49,032,194	1,366,979	4,426,426	4,897,415	5,764,796	5,915,535	4,619,047	9,497,149
Average absolute volume ($m)	690	87	189	206	210	236	190	186
Cumulative absolute volume ($m)	345,217	13,398	28,480	31,167	31,899	35,820	28,816	59,563
Average relative volume	0.009	0.011	0.009	0.009	0.007	0.009	0.009	0.009
Cumulative relative volume	4.422	1.645	1.316	1.295	1.111	1.376	1.307	2.817
Developed world								
Average market capitalisation ($m)	71,735	6,269	21,773	23,493	27,428	27,826	22,681	21,581
Cumulative market capitalisation ($m)	140,571,683	3,805,056	12,978,101	14,119,157	16,574,186	16,825,377	13,667,264	26,914,294
Average absolute volume ($m)	445	53	127	133	137	151	126	121
Cumulative absolute volume ($m)	875,123	32,115	75,679	80,605	82,942	91,885	76,464	151,433
Average relative volume	0.007	0.008	0.007	0.007	0.006	0.007	0.007	0.007
Cumulative relative volume	14.324	4.855	4.226	4.094	3.578	4.291	4.173	8.655
Emerging markets								
Average market capitalisation ($m)	18,718	1,458	4,284	4,399	4,354	4,816	3,922	3,872
Cumulative market capitalisation ($m)	12,932,939	308,409	892,739	928,367	915,362	1,014,946	827,831	1,670,509
Average absolute volume ($m)	75	6	16	16	14	18	13	14
Cumulative absolute volume ($m)	51,852	1,314	3,354	3,452	2,919	3,763	2,763	5,997
Average relative volume	0.005	0.005	0.004	0.004	0.003	0.004	0.004	0.004
Cumulative relative volume	3.537	0.962	0.885	0.926	0.728	0.893	0.832	1.780

Analytics are calculated over the period 31 December 2006 to 31 December 2016. All average measures are weighted averages for the corresponding indices. All cumulative measures are weighted averages multiplied by the number of stocks in the index. The average absolute volume is the weighted average ADTV of all the stocks in the index. The ADTV of a stock is calculated as the median of the quarterly average daily dollar traded volume over the last four quarters. The traded volumes are calculated from the three most liquid listings of the stock. In calculating the relative volume, the ADTV of a stock is divided by its capitalisation and then this ratio is used in calculating the weighted average of the index.

mid-cap indices have the highest days-to-trade and high profitability indices have the lowest days-to-trade. Amihud measure also shares the same trend as that of other investability measures.

Conclusion

Smart beta strategies do incur additional costs compared to cap-weighted indices. A reasonable expectation from an investor's perspective is that providers should disclose the level of costs generated by their strategies in order to provide information on net returns. However, providers typically fail to make explicit adjustments for implementation costs and merely report gross returns, leaving it to other market participants to figure out what the costs are like. Scientific Beta addresses this gap by introducing a suite of analytics developed specifically to enable investors to assess the investability of smart beta indices. These analytics show that while it is true that smart beta strategies incur additional costs, carefully designed indices with a focus on investability do generate relative returns in excess of the costs. It is worth emphasising that none of these analytics relies on any proprietary data. They are computed using readily available low-frequency market data with no major computational overhead and a transparent methodology that can be easily replicated and so there is no reason why smart beta index providers should delegate the investability assessment to the market participants, especially when it is one of the key elements in smart beta investment decision-making. In order to improve transparency and enable investors to better assess the smart beta strategies, these metrics should be widely used in the industry.

3. Days-to-trade and price impact

31 December 2006–31 December 2016

	Broad cap-weighted	High Factor Intensity Diversified Multi-Strategy indices						
		Mid cap	Value	High momentum	Low volatiity	High profitability	Low investment	Multi-Beta Multi-Strategy (6 Factor) EW
US								
Maximum days-to-trade (as of last rebalancing date)	0.23	2.29	2.32	2.48	2.32	2.05	2.38	1.32
Effective days-to-trade	0.01	0.39	0.21	0.27	0.18	0.15	0.19	0.17
Amihud illiquidity ratio	0.11	0.21	0.13	0.12	0.11	0.11	0.12	0.13
Developed world								
Maximum days-to-trade (as of last rebalancing date)	0.38	6.76	5.18	5.05	5.59	3.97	4.75	2.64
Effective days-to-trade	0.04	1.54	0.77	0.88	0.73	0.61	0.67	0.58
Amihud illiquidity ratio	0.72	2.23	1.47	1.18	1.26	1.07	1.29	1.42
Emerging markets								
Maximum days-to-trade (as of last rebalancing date)	0.88	10.52	7.75	6.71	8.83	6.16	7.53	4.60
Effective days-to-trade	0.09	2.28	1.07	1.24	1.22	0.73	1.08	0.87
Amihud illiquidity ratio	9.97	31.55	28.93	19.61	26.21	17.92	29.18	25.57

Analytics are calculated over the period 31 December 2006 to 31 December 2016. In calculating days to trade (DTT) of an index (both effective and maximum DTT), we assume an investment of $3bn in a global cap-weight index in June 2016. The effective DTT is reported as the quarterly average of index DTT for the past 10 years/40 quarters, whereas maximum DTT is reported only for the latest quarter. In both cases, every quarter, first we calculate the DTT of a stock assuming daily trading volume used up is 10% and then the DTT of a portfolio is calculated as the 95th percentile value. In the case of effective DTT, we only consider changes in the weight of a stock, whereas in the case of maximum DTT, we use the weight of a stock instead of the change in weight at rebalancing. The Amihud illiquidity ratio of an index is the weighted average Amihud measure of stocks, which in turn is calculated as the ratio of absolute daily returns in local currency to the daily dollar traded volume. Source: www.scientificbeta.com.

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Debating the merits of 'top-down' and 'bottom-up' approaches to multi-factor index construction

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Multi-factor index providers debate the respective merits of the 'top-down' and 'bottom-up' approaches to multi-factor equity portfolio construction.

'Top-down' approaches assemble multi-factor portfolios by combining distinct sleeves for each factor. 'Bottom-up' methods build multi-factor portfolios in a single pass by choosing and/or weighting securities by a composite measure of multi-factor exposures.

We contrast the claims of the proponents of bottom-up approaches with relevant findings in the academic literature.

We review general insights from the literature on return estimation and factor models that are relevant for multi-factor portfolio construction.

We discuss recent literature that specifically addresses issues with bottom-up portfolio approaches.

Multi-factor index providers have debated the respective merits of the 'top-down' and 'bottom-up' approaches to multi-factor equity portfolio construction. 'Top-down' approaches assemble multi-factor portfolios by combining distinct sleeves for each factor. 'Bottom-up' methods build multi-factor portfolios in a single pass by choosing and/or weighting securities by a composite measure of multi-factor exposures.

The 'top-down' approach is simple and transparent and investors can control allocations across factors easily. Being typically assembled from reasonably diversified factor sleeves, 'top-down' multi-factor portfolios avoid being concentrated in a few stocks. 'Bottom-up' portfolios have been used to concentrate portfolios in 'factor champions', where one emphasises stocks that score highly on average across multiple factors. This allows interactions across factors to be taken into account and avoids diluting exposures (such as diluting exposure to value when tilting to high profitability).

It has been argued that bottom-up approaches produce additional performance. However, studies of bottom-up approaches that document increased returns are typically based on selected combinations of factors (Bender and Wang [2016], Clarke et al [2016], Fitzgibbons et al [2016], FTSE [2016]) and short samples (FTSE [2016]). They also do not test for significance or robustness, and do not scrutinise risks, stability of exposures, and implementation issues such as heightened turnover. Moreover, in a recent study, Amenc et al (2017) have shown that accounting for the cross-sectional interaction effects of factors does not necessarily require a bottom-up approach but can be addressed in a suitably designed top-down framework.

Against this backdrop, we contrast the claims of the proponents of bottom-up approaches with relevant findings in the academic literature. In the first section we review general insights from the literature on return estimation and factor models that are relevant for multi-factor portfolio construction. In the second section we discuss recent literature that specifically addresses issues with bottom-up portfolio approaches.

Does it make sense to account for fine-grain differences in factor exposures?

A key idea behind bottom-up approaches is precisely to account for stock-level differences in terms of exposure to multiple factors. While it is understandable that computational technicians will have a tendency to try to account for factor exposures with the highest possible precision, it is worth considering insights from finance. There are two findings in empirical asset pricing that question the relevance of the type of over-engineering present in bottom-up approaches.

Stock-level estimates are noisy

Empirical evidence on factor premia overwhelmingly suggests that the relationships between factor exposures and expected returns do not hold with a high level of precision at the individual stock level. Indeed, factor scores are used as proxies for expected returns, which are notoriously difficult to estimate and inherently noisy at the stock level (see Merton [1980] and Black [1993a]).

Rather than trying to determine differences in returns between individual stocks, researchers have created groups of stocks and tested broad differences in returns across these. This 'portfolio method' ensures robustness by ignoring stock-level differences and refraining from modelling multivariate interactions. For this reason, studies that document factor premia (such as Fama and French [1993]) rely on portfolio-sorting approaches. In particular, Black (1993b, p77) emphasises "I am especially fond of the 'portfolio method' [...]. Nothing I have seen [...] leads me to believe that we can gain much by varying this method."

There is ample evidence suggesting that factor characteristics do not provide an exact link with individual stock returns (see Cederburg and O'Doherty [2015]) and often this is not even monotonous (see Patton and Timmermann [2010]). Thus fine-grain differences in factor exposures may not translate into return differences.

To illustrate the lack of precision in the relationship between factor exposure and returns, we provide results for fine-grain portfolio sorts. In particular, we first sort quintile portfolios by factor characteristics (such as book-to-market for 'value'), and then in a second sort each quintile is again subdivided into sub-quintiles according to the same factor score. If the relationship between factor exposure and returns were highly precise, even the second sort for stocks with broadly similar characteristics should lead to meaningful return differences. To be more specific, even when looking at stocks in the same book-to-market quintile, the distinction by sub-quintile in a second sort should lead to a positive value premium being observed for those stocks that are more value-oriented (higher book-to-market ratio) within their respective quintile. However, as can be seen from figure 1 on page12, the sub-quintile premia are negative in most cases. Especially in the winner quintile (Q5), distinguishing between stocks based on factor scores does not add any value. In fact, for four out of the six factors we analysed, selecting the highest-exposure stocks among the top quintile stocks leads to lower returns than selecting the stocks with the relatively lowest exposure in the top quintile. In other words, among stocks with high exposure to a given

1. Inter-quintile premiums of each factor

31 Dec 1975–31 Dec 2015 (Q5–Q1)	Size	Value	Momentum	Low volatility	Low investment	High profitability
Q1 (Low exposure stocks)	0.53%	0.11%	6.23%	5.50%	9.06%	4.38%
Q2	–0.91%	–0.31%	4.45%	1.20%	1.11%	–3.08%
Q3	–0.77%	–0.39%	–1.58%	1.58%	1.63%	2.49%
Q4	–1.18%	2.68%	0.25%	0.75%	–0.05%	0.93%
Q5 (High exposure stocks)	**–0.38%**	**–0.06%**	**1.62%**	**–0.94%**	**1.61%**	**–0.28%**

Analysis is based on daily total returns in US dollars from 31 December 1975 to 31 December 2015 based on the 500 largest stocks in the US. For each factor the universe is divided into 5 by 5 double sorting based on the corresponding factor score is carried out and 25 equal weighted portfolios are formed. The difference in returns between the fifth and first quintiles after the second sort across each quintile from the first sort is reported.

2. Bottom-up vs top-down approaches – Sharpe ratio comparisons

	Bottom-up portfolios with higher Sharpe ratio	
78 portfolios (3 construction methods * 26 possible combinations of 5 factors)	Number of portfolios	Statistically significant at 5% when adjusted for multiple hypotheses
Difference in Sharpe ratio	67 (86%)	10 (13%)
Difference in Sharpe ratio at similar relative risk	35 (45%)	0 (0%)

The table presents the summary of results discussed in Leippold and Rüegg (2017) based on a long history of US stock returns (1963 to 2014).

factor (top quintile stocks), making a finer distinction between those that are most strongly exposed and those that are relatively less strongly exposed does not lead to higher returns. This clearly shows that even though the risk premium appears in broadly diversified portfolios, it disappears if we start accounting for differences at the stock level or create very narrow portfolios according to precise differences in exposures.

Single-factor relationships may break down at the multi-factor level

While there is ample evidence that portfolios sorted on a single characteristic are related to robust patterns in expected returns, such patterns may break down when incorporating many different exposures at the same time. For example, Asness (1997, p29 and p34) observes: "Value works, in general, but largely fails for firms with strong momentum. Momentum works, in general, but is particularly strong for expensive firms." As a result "increasing both momentum and value simultaneously has a significantly weaker effect on stock returns than the average of the marginal effects of increasing them separately". This weakening would impact securities favoured by composite scoring methods.

A more drastic failure is discussed by Stambaugh, Yu and Yuan (2015). The authors show that, even though the low volatility anomaly exists in the broad cross-section of stocks, low volatility stocks actually underperform when considering only stocks that rank well on a composite multi-factor score. Building bottom-up multi-factor portfolios on the basis of factors that have been documented in a top-down framework thus lacks relevance.

Ultimately, engineering multi-factor portfolios under the assumption of a deterministic dependence of returns on security-level multi-factor scores means exploiting information which is not reliable.

Could the backtest performance of 'bottom-up' approaches be overstated?

A backtest is a simulation of a portfolio performance as if it were implemented historically. It is not rare to find strategies that provide stellar performance in backtests but fail to deliver robust live performance. There are several reasons for such a lack of robustness. Firstly, backtests are sensitive to the sample period of the tests. This problem arises simply because returns are highly sample-specific. Secondly, the results of backtests could be contaminated by data mining and over-fitting. Lo and MacKinlay (1990) wrote that, "[...] the more scrutiny a collection of data is subjected to, the more likely will interesting (spurious) patterns emerge". If one generates and tests enough strategies, one will eventually find a strategy that works very well in the backtest. Over-fitting occurs when more and more degrees of freedom are added to the model until the model might actually be capturing sample-specific noise rather than structural information. Over-fitted models tend to fail miserably out-of-sample.

The bottom-up approach to multi-factor investing has opened up a platform for computational technicians to come up with several possibilities for selecting and weighting factor metrics for multivariate composite scores. Such post hoc combinations exacerbate data-mining problems by introducing over-fitting and selection biases (see Novy-Marx [2016]). Knowing that the bottom-up approaches are by design prone to selection bias, an important question worth exploring is whether the claims of bottom-up proponents could be due to statistical flukes. A simple way to do that is by adjusting the results for the inherent biases. The discussion below explores this question in detail by summarising results from a recent study (Leippold and Rüegg [2017]).

Even though the multiple testing bias has been extensively analysed in the literature (see McLean and Pontiff [2016], Harvey, Liu, and Zhu [2016] and Baily and Lopez de Prado [2014]), studies claiming that bottom-up approaches provide better risk-adjusted returns than top-down approaches do not account for this issue (see Bender and Wang [2016]). Moreover, tests are done on short time periods such as 15 years (see FTSE [2016]) while a reasonable empirical assessment of factor investing approaches warrants a substantially longer time period (40 years or more) to account for the cyclical nature of risk factors.

A recent study (see Leippold and Rüegg [2017]) re-assesses claims that a bottom-up approach to multi-factor portfolio construction leads to superior results. When applying proper statistical robustness checks, and adjusting for relative risk, they find that there is no such superiority.

Leippold and Rüegg account for the fact that there are numerous variations one could employ to conduct such tests and any reported superiority of the bottom-up approach could be the result of picking a favourable combination that happens to 'work' simply due to chance. The authors test a large variety of factor combinations and portfolio construction methods, and compare the bottom-up and top-down approach in each case. They use advanced statistical tools to adjust for the fact that apparently significant benefits will easily result as a fluke if the number of combinations is large enough.

This analysis shows that there is no evidence that bottom-up approaches perform better than the corresponding top-down approaches. Thus, the findings reported by promoters of bottom-up approaches do not withstand rigorous analysis and could instead be explained by the choice of a particular selection of factors, and failure to adjust for the data-mining possibilities offered for such analysis.

Figure 2 presents a summary of the results. The authors created 78 different multi-factor portfolios using all possible combinations of up to five popular factors (value, momentum, investment, profitability and low volatility) and three different portfolio construction methods. Only 13% of the possible variations lead bottom-up portfolios to have significantly higher Sharpe ratios than top-down approaches when adjusting for multiple testing. Moreover, when adjusting the top-down portfolios to match the levels of relative risk of the bottom-up portfolios, none of the bottom-up portfolios has significantly higher Sharpe ratios than their top-down counterpart. This finding invalidates the claims of superiority made by proponents of bottom-up approaches.

Thus, while some claim that bottom-up portfolios generate superior performance, a thorough analysis shows that the evidence does not support such claims. For investors, it is important to keep in mind the potential data-mining pitfalls associated with backtests. Leippold and Rüegg (2017, p24) note: "Given the increasing computational power for conducting multiple backtests and given the fact that financial institutions have incentives to deliver extraordinary results, it is crucial to apply the most advanced statistical testing frameworks. Ignoring the available tools can lead to hasty conclusions and misallocation of capital to investment strategies that are false discoveries."

While providers are entitled to rely on short-term backtests to conclude on the superiority of their approach, investors would be well advised to consider the findings in the academic finance literature and to use advanced statistical tools when they evaluate the benefits of bottom-up approaches.

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Diversification within an equity factor-based framework

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An understanding of the design choices underlying multi-factor products is crucial if investors are to avoid outcomes that may ultimately disappoint them.

These design choices include: factor selection, starting universe, multi-factor construction approach, stock weighting scheme, factor weights, regional allocation and currency exposure.

Using evidence and beliefs, we outline a 'blank-sheet-of-paper' approach to design a particular strategy that places a heavy emphasis on diversification at the factor, region, sector and stock level.

This leads to considered objectives for portfolio return, risk and diversification which are able to be clearly messaged to investors.

As factor-based investing has increased in popularity since the financial crisis, so has the number of products available for investors to choose from. Underlying each of these products is a set of design choices whether they are explicitly or implicitly made. For investors we believe it is critically important that they understand these design choices in order to assess how a strategy is likely to perform in the different environments it will invariably face.

We outline a multi-factor equity strategy where a 'blank-sheet-of paper' approach is taken to the following design choices which are explicitly considered and incorporated within the end strategy:

- Factor selection;
- Starting universe;
- Multi-factor construction approach;
- Stock weighting scheme;
- Factor weights; and
- Regional weights and currency exposure.

Having explicit consideration of these areas facilitates clear messaging to investors with respect to the relevant objectives for the strategy and its characteristics. Underlying our strategy's philosophy is a belief in the power of diversification which can be shown not only to reduce risk but to improve geometric returns[1]. Diversification can occur at different levels and is pivotal to the strategy's construction.

Below we highlight the design choices that we make in designing the strategy and elaborate on the evidence and beliefs that underpin these choices.

Factor selection

Through our research and investment experience we have developed beliefs on the merits of different factors. While a vast number of factors are documented in academic literature, with over 300 found in one study[2], there are relatively few that have an established body of academic research associated with them. Those that we find to be covered more consistently include: value, low volatility, quality, momentum and size.

These correspond closely to the ERI Scientific Beta range of factors available. The key distinction to be made is with regard to the quality factor. ERI Scientific Beta considers quality to be composed of two separate and distinct factors, namely high profitability and low investment, which is in line with Fama and French (2014). We agree with this assessment though we are cautious in giving these two factors as much weight as more established factors. High profitability and low investment have only been published in the academic literature in this century while factors such as value, momentum, size and low volatility all have papers associated with them from the previous century. As such, our confidence in high profitability and low investment is reflected through an adjustment such that each receives half-weight. In essence, these two factors equally-weighted combine to form a single 'quality' factor.

Additionally, we have a prior belief that cross-sectional momentum (ie, momentum at the stock level) is difficult to capture through regularly-rebalanced indices and may induce additional turnover without significant additional benefit, particularly within a multi-factor framework. Published papers by Koraczyk and Sadka (2004) and more recently Novy-Marx and Velikov (2016) support the belief of limited capacity for momentum strategies prior to alpha erosion though a working paper by Frazzini, Israel and Moskowitz (2015) challenges this wisdom. However, this latter paper uses a proprietary dataset which cannot be scrutinised. Where momentum is to be used in portfolios, our general preference is to target time-series momentum involving futures contracts rather than cross-sectional momentum involving individual stocks with the aim to reduce the transaction costs of trading momentum (and indeed Pedersen, Moskowitz and Ooi [2012] present

1 Humble and Southall (2014).
2 Harvey et al (2016).

evidence of time-series momentum's ability to completely explain cross-sectional momentum in equities). In our testing we retain momentum as a possible factor for consideration though it faces a higher hurdle for inclusion based on the prior belief.

As we will note later, the Scientific Beta High Factor Intensity (HFI) indices, which we have chosen to use, incorporate a filter which removes stocks with poor multi-factor scores. Momentum is an input into the multi-factor score which means that stocks that score poorly on momentum, all else equal, are more likely to be filtered out. We feel that by removing stocks with poor momentum rather than focusing on stocks with good momentum, this enables us to incorporate the factor in an efficient way.

Our aim in factor selection is to have enough factors such that factor diversification is effective though crucially we must have a high level of belief in these factors.

Starting universe

Given our aim is to construct a global multi-factor equity strategy, the key question with respect to the starting universe is whether to include emerging markets or restrict the choice to developed markets where there is already a significant body of research on the existence of factors. We find evidence of all the main factors above working in emerging markets as listed in figure 1[3] and as such we include this region within our universe. This improves our ability to diversify across the markets of more countries, many of which are less co-integrated with developed markets.

Multi-factor construction approach

A key debate going on within factor investing circles surrounds the issue of multi-factor portfolio construction: whether to go 'top-down' or 'bottom-up' with the factor exposures[4]. The top-down approach allocates to factors as individual building blocks. For example, a top-down multi-factor strategy might have allocations to a value portfolio, a quality portfolio and a low volatility portfolio (where each of these portfolios contains stocks that score strongly on their respective characteristics).

In contrast, the bottom-up approach to multi-factor investing gives each stock in the universe a score on each of the desired factors. These individual factor scores are then combined into an overall multi-factor score for each security in the universe. This composite score is then used to derive a weight in the multi-factor portfolio. There is variation in methodology across different bottom-up strategies.

1. Factor premiums

Factor (long/short)	Sample	Period	Premium	Source
Value	Emerging markets	1990–2011	1.15% (monthly mean)	Cakici, Fabozzi and Tan (2013)
Momentum	Emerging markets	1990–2011	0.86% (monthly mean)	Cakici, Fabozzi and Tan (2013)
Size	Emerging markets	1990–2011	0.28% (monthly mean)	Cakici, Fabozzi and Tan (2013)
Low volatility	Emerging markets	1999–2012	2.10% (annual mean)	Blitz, Pang and Van Vliet (2013)
Low investment	Emerging and developed markets	1982–2010	6.18% (annual mean)	Watanabe, Xu, Yao and Yu (2013)
High profitability	Emerging markets (Europe)	2002–14	0.71% (monthly mean)	Zaremba (2014)

The table presents the factor premium identified for each of six factors from four studies. The factor premium in each case is defined as the difference in returns between a portfolio that contains the top quintile of stocks exposed to the factor and a portfolio that contains the bottom quintile (ie, the least exposed to the factor). These are effectively long/short factor premiums. Note that for the Watanabe, Xu, Yao and Yu study, portfolios based off top and bottom terciles, quintiles and deciles are used depending on whether there are 30, 50 or 100 stocks respectively for each country-year.

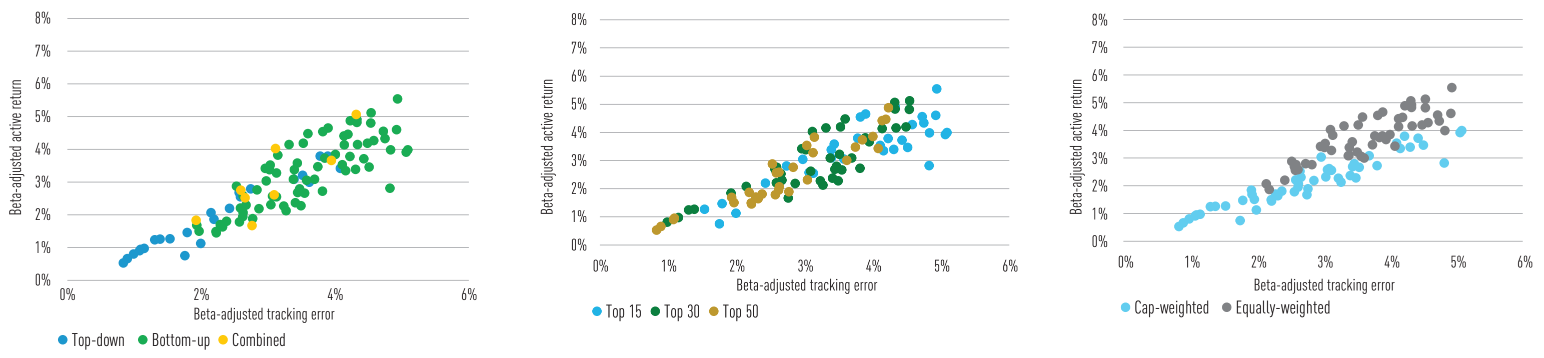
Additionally, ERI Scientific Beta in 2017 introduced its new range of High Factor Intensity (HFI) multi-factor indices which retain the overall top-down structure used within its original Multi-Beta Multi-Strategy range of multi-factor indices though it adds a bottom-up filtering process applied within each factor sleeve, as described in Amenc et al (2017). We see this as a way of effectively synthesising the top-down and bottom-up approaches. It preserves the simplicity and transparency of the top-down approach but accounts for the cross-factor interaction which, until now, has only been captured by bottom-up approaches. The key difference from other providers we find is in its use of the bottom-up part of the process. Here ERI Scientific Beta focuses on eliminating stocks with poor multi-factor scores rather than adding weight to stocks with strong multi-factor scores – a method which is prevalent among most pure bottom-up approaches.

In order to understand the difference between the bottom-up approach, the top-down approach and the two step-filtering approach present in the HFI methodology (from here on referred to as the 'combined' approach), we conduct our own independent empirical research to understand strategy characteristics. One of the key challenges in comparing approaches across index providers is due to differences in factor definitions, stock weighting schemes or stock universes, among others. Hence it is important to construct strategies using a uniform set of inputs apart from the multi-factor construction approach in order to create a true 'apples-to-apples' comparison.

When examining results, two measures of risk-adjusted return we consider are beta-adjusted information ratio (ie, beta-adjusted active return over beta-adjusted tracking error[5]) and Sharpe ratio. We find that the combined

3 Adapted from Shirbini (2016).
4 See Fitzgibbon et al (2016) and Bender and Wang (2016).
5 Beta-adjusted active return is what is known otherwise as 'Jensen's alpha' or 'ex-post alpha' as described in Jensen (1967) using the Capital Asset Pricing Model (CAPM) beta to adjust active returns. Beta-adjusted tracking error is a related statistic equal to the volatility of the beta-adjusted active returns. We prefer the beta-adjusted information ratio over the standard information ratio as it does not automatically penalise strategies with beta less than 1 (which is a desirable characteristic for some investors).

2, 3 & 4. Risk/return charts



The risk/return charts above show the results of beta-adjusted active return and beta-adjusted tracking error for various multi-factor portfolios using monthly returns. These portfolios are composed of: (1) different multi-factor construction approaches, (2) different stock weighting schemes and (3) different sector/region neutrality constraints.

The multi-factor construction approaches include: (i) a top-down approach based on top *x*% selection within each factor sleeve where *x* = 15, 30 and 50, (ii) three different bottom-up approaches including geometric S-score multi-factor scores, arithmetic S-score multi-factor scores and average factor rank multi-factor scores*, all based on top *x*% selection for the overall portfolio where *x* = 15, 30 and 50 and (iii) the combined approach which uses a top 50% initial selection per the top-down approach in (i) followed by a top 60% selection based on average factor rank multi-factor scores within each factor sleeve. The two stock weighting schemes tested are capitalisation-weighting and equal-weighting. Portfolios incorporating region-neutrality, sector-neutrality as well as region and sector-neutrality are examined alongside the unconstrained version. This leads to 104 different multi-factor portfolios being formed (13 multi-factor construction approaches × two weighting schemes × four portfolio constraint options). We note though that our list of approaches tested is far from exhaustive and indeed only scratches the surface with some of the most common found within the industry.

Our dataset uses information for a global universe of stocks (including developed and emerging market stocks) between March 2002 and December 2016. The benchmark is a cap-weighted portfolio including all stocks in the universe. The factors we include are value (book-to-price), low volatility (based on one-year daily returns), momentum (last 12 months return omitting the most recent month) and quality (which is an equally-weighted combination of high profitability – gross profits-to-assets definition - and low investment – based on three-year asset growth). These four factors (value, low volatility, momentum and quality) are given equal weight. We form portfolios that are semi-annually rebalanced at the end of February and the end of August.

approach which features in the HFI methodology stacks up well against the various other methodologies. The combined approach with region-neutral formation and equal-weighting, which maps closest to the methodology in HFI indices, has a beta-adjusted information ratio in the top decile and a Sharpe ratio in the third decile. However we would de-emphasise the importance of this empirical testing. We see the testing as validating our belief that the combined approach is an efficient implementation rather than it driving our decision.

Bottom-up strategies are seen to carry higher beta-adjusted tracking errors which are generally commensurate with higher beta-adjusted returns though this link weakens for concentrated approaches (ie, top 15% selection[6]). Overall on this risk-adjusted return measure we find that there is generally a linear relationship between risk and return for all strategies except those with particularly high beta-adjusted tracking errors (which mostly correspond with concentrated portfolios).

In terms of stock weighting schemes, we find that equal-weighted strategies dominate cap-weighted strategies with this being robust to examining time periods when the size factor produced a zero return. This would indicate that the performance of the size factor may not have been the only driver of the performance differential but could be due to the effects of better diversification and lower stock-specific risk for equal-weighted strategies.

Also, we notice that bottom-up strategies have tended to carry a low beta bias over the time period. This finding is similar to that of Jivraj et al (2016) who also look at multi-factor construction approaches that include the low volatility factor for a US universe between January 2003 and July 2016.

Additionally, we find that while region-neutrality (formed using 11 regional building blocks akin to the approach taken by ERI Scientific Beta in its methodology) leads typically to improvements in beta-adjusted information ratio relative to a global approach to stock selection, this also typically leads to small declines in Sharpe ratio. We find little support for either sector neutrality or region-and-sector neutrality on a performance basis where sector neutrality is achieved through a re-scaling process back to market-cap sector weights (at the global level for sector-neutral and within region for region-and-sector neutral). We also note the higher turnover of these strategies, particularly for region-and-sector neutrality.

While we do not believe that the time period used in our research is long enough to make definitive conclusions, we would argue that it still provides some level of insight. We acknowledge the results in Amenc et al, who use US stock data over the period 1975 to 2015 to confirm the robustness of the Scientific Beta HFI approach relative to a concentrated bottom-up approach. Similarly, Leippold and Rüegg (2017), who use US stock data from 1963 onward, find a similar pattern as us with regard to the low beta bias of bottom-up strategies that include the low volatility factor while also finding similar levels of risk-adjusted return between top-down and bottom-up approaches.

Overall, having undertaken the independent research above, our results seem to favour the combined approach which aligns with the methodology within ERI Scientific Beta HFI indices. This validates our belief that the combined approach is an efficient way of integrating bottom-up and top-down approaches. As such, we decide to use indices within this range to implement our multi-factor strategy.

Stock weighting scheme

When considering how to weight individual stocks after the stock selection process, our goal is to seek diversification such that stock-specific risk is reduced. When deciding what weight to give assets, we like to consider both capital weights as well as risk weights. Diversifying by capital weights corresponds to the ERI Scientific Beta maximum deconcentration stock weighting scheme while diversifying by risk weights corresponds to the ERI Scientific Beta diversified risk weighted stock weighting scheme. As such we use an equal-weighted combination of these two weighting schemes. This leads to a significant reduction in stock-level concentration relative to cap-weighted indices[7], which is a common aim for many investors.

By seeking this diversified stock weighting scheme, we note that we also by proxy achieve more diversified sector weights. This has the effect of reducing the influence of the largest sectors which could be susceptible to periods of over-valuation[8].

Factor weights

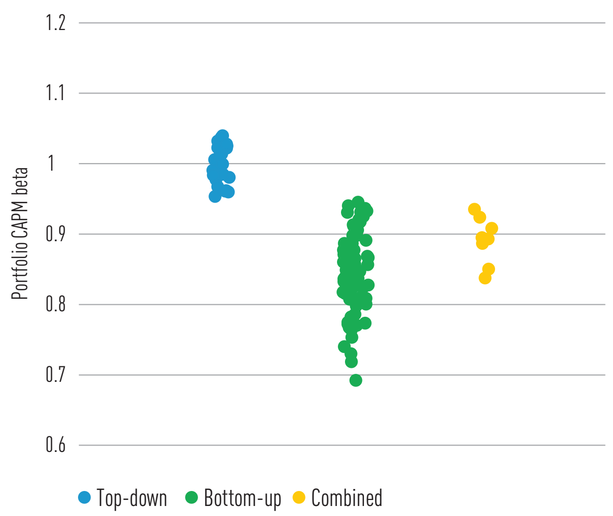
Our primary objective when setting factor weights is to seek diversified factor exposure. This means that we want to ensure that we are carrying significantly positive and relatively balanced exposures to the factors we are targeting

6 Note that the combined approach leads to the equivalent of a top 30% selection (ie, a top 50% selection followed by a top 60% selection).

7 As an indication of the reduction, the percentage weight in the 20 largest stocks in our strategy in June 2017 was about 5% while this was about 15% for a representative global market-cap equity index.

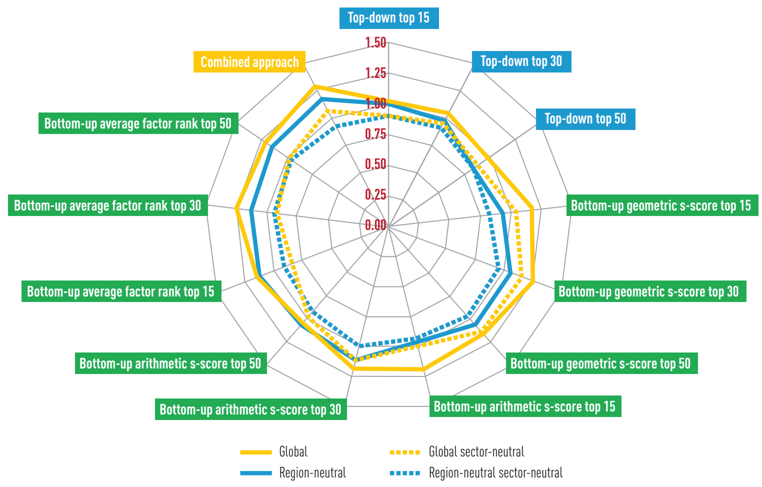
8 For example, the information technology sector during the 'dot-com' bubble came to represent about 30% of the S&P 500 at one point.

5. Portfolio CAPM beta



The figure above shows the portfolio CAPM beta for the 104 different portfolios referred to in figures 2, 3 and 4 split by multi-factor construction approach. The CAPM beta is defined as the slope of the regression of portfolio returns on the market factor.

6. Beta-adjusted information ratio: equally-weighted portfolios



The figure above plots the beta-adjusted information ratio for the 52 equally-weighted portfolios referred to in figures 2, 3 and 4 across 13 different multi-factor construction approaches and four portfolio constraint options.

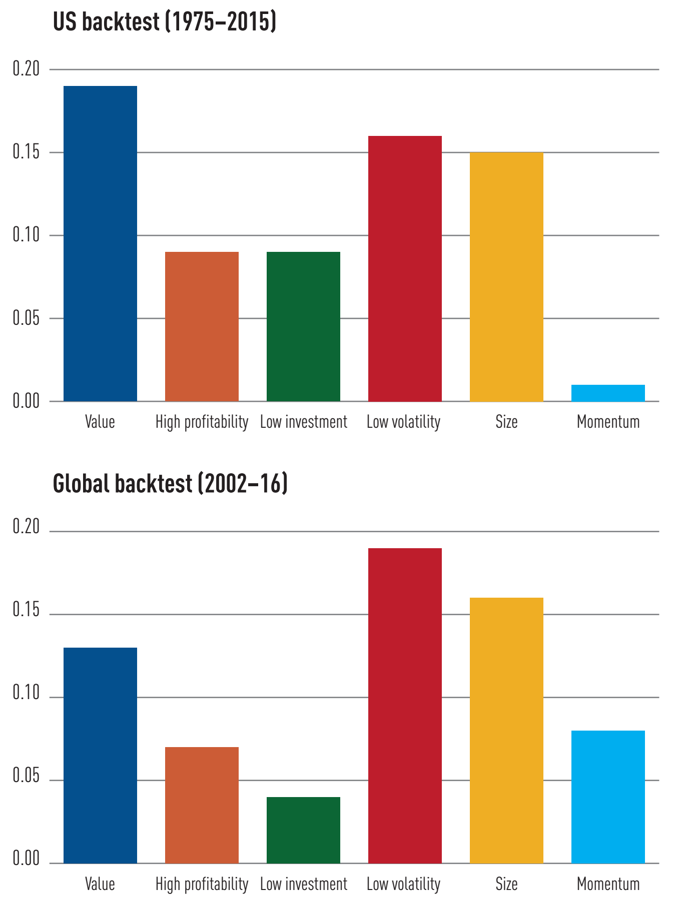
through the economic cycle. Our starting point is to test equal factor weights and if then there is a need to deviate from this position we would do so. However, as the ERI Scientific Beta HFI methodology explicitly accounts for cross-factor interactions, we expect factor balance to naturally occur.

Additionally, as we are opting for a diversified stock weighting scheme, we recognise that this introduces a significant amount of size factor exposure itself and an explicit allocation to the factor would lead to an imbalance that would go against our objective. We test different factor weights over two time periods: for a US stock universe from 1975 to 2015 and for a global stock universe from 2002 to 2016. We see broadly similar overall return and risk statistics across different combinations of factors including those that held momentum and size factors though critically there are differences when it comes to factor exposures.

We notice a heavy imbalance in factor exposures when including the size factor which matches our initial intuition. When looking at the momentum factor exposure over the US long-term backtest, while still being statistically significant, it is markedly lower in magnitude relative to the other factors (considering high profitability and low investment as a single factor). This means that possibly momentum has indeed been more difficult to capture over the long term. This gives us sufficient reason to exclude momentum as we have not seen much evidence to challenge our prior belief.

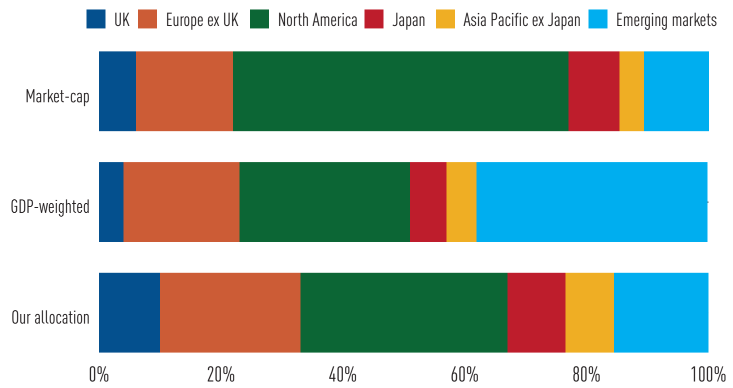
As a result of excluding explicit allocations to momentum and size, we are left with equal weighting value, low volatility and quality factors (where

7. Factor exposure



The charts above represent a measure of factor exposure for our chosen strategy. These factor exposures are defined as the factor regression coefficients from a seven-factor model that includes the market factor alongside six long/short factor portfolios (value, high profitability, low investment, low volatility, size and momentum). The results for these six factors only are shown. The first chart uses a US universe of stocks between the period December 1975 and December 2015 while the second chart uses a global universe of stocks (including emerging markets) between June 2002 and December 2016. Weekly returns are used in each case. The calculation has been performed by ERI Scientific Beta.

8.Regional allocations



The chart above shows the weights of different regions based on: (i) market capitalisation using data from ERI Scientific Beta, (ii) GDP based on data from the World Bank and (iii) our target allocation.

quality itself is an equal-weighted combination of high profitability and low investment). We note that we achieve very good factor balance across the factors targeted in the US long-term backtest. The balance is not as good in the shorter-term global backtest, though we feel this to be still quite reasonable. All factor exposures across the two backtests are significant at the 1% level except for momentum in the US long-term backtest. While momentum is positive and significant in the shorter-term global backtest (even though it is not explicitly targeted), we would not expect this to be the case over all time periods given the result of the longer-term US test.

This leaves us with diversified factor exposure, which added to diversification at stock and sector level, all enable us to reduce the risks of particular factors, stocks or sectors performing poorly.

Regional weights and currency exposure

The final layer of diversification we seek is with regard to the strategy's regional weights and currency exposures. Often a multi-factor strategy's regional weights are an artefact of the stock selection process. We believe this risks the introduction of unintended regional bets. An explicit regional allocation process can alleviate the issue.

When deciding upon regional allocations, we split the global equity universe into six distinct regions: UK, developed Europe ex UK, North America, Japan, developed Asia Pacific ex Japan and emerging markets[9].

Individual regional weights can be chosen in many alternative ways. In addition to an equally-weighted allocation, two of the most intuitive alternatives include market-cap weighting and GDP weighting.

The market-cap weighting approach is widely adopted in index-based investing due to its straightforward implementation, removing the need to rebalance among regions[10]. However, we recognise that one of the key aims of investors when considering investments in this area is to avoid links to market-cap weighting as it can re-introduce the sensitivity to company valuations. As valuations of an individual stock or a group of stocks within a region increase, this would drive higher the weight of the region where these stocks are listed. Furthermore, such an approach results in a large concentration in North America, with nearly 60% weight in the region at present.

Nevertheless, while acknowledging its limitations, the market-cap weights of individual regions represent their importance in financial markets and as such they remain a dimension worthy of consideration. However, we believe it should be accounted for in conjunction with the regions' economic significance that is reflected in their GDP. The GDP weighted approach breaks the link between country weightings and market-cap size, hence reducing the sensitivity of regional exposures to changes in market sentiment. Consequently, the weight of larger emerging market economies like China will be higher and the overall regional exposure could be significantly different from the conventional market-cap benchmark.

The regional allocation we choose for our global multi-factor equity strategy aims to reflect both the economic and the financial significance of individual regions to provide a more diversified exposure that is not overly reliant on any single region. To enhance that diversification even further we marginally increase the weight of those regions that are less correlated with the home market. For a UK investor, this would normally mean a positive adjustment to Asia Pacific, including Japan, and emerging markets on a stand-alone basis. For investors based elsewhere, for example in the eurozone, these adjustments would be different. We also ensure that the chosen allocation does not result in a concentrated exposure to the politics of a specific country. That on a stand-alone basis would lead to a reduction in weight of regions such as the UK and Japan, and to a slightly lesser degree, North America. Finally we also consider governance standards to determine whether investors get the returns they earn for taking the equity risk in a particular region. Overall, our approach results in a more balanced regional allocation which further reduces the strategy's concentration risk (figure 8).

Interestingly we note that by lowering the weight of the North American region we are able to increase stock-level diversification as measured by the effective number of stocks[11]. Our multifactor strategy using market-cap regional weights results in an effective number of stocks figure of 657 while the same strategy using our regional weights has a figure of 852 as of December 2016. This increase is possible due to the high average stock weight for the North American universe relative to other regions. By moving weight to other regions, this leads to greater diversification at the stock level as measured by effective number of stocks.

In addition to the regional allocation, for a UK investor, we would hold currency exposures which hedge 50% of the overseas (ie, non-GBP) developed

9 Note that this is similar though different to the 11 ERI Scientific Beta regions (eight developed, three emerging) that were used in our multi-factor construction approach research detailed previously.

10 Although rebalancing would still occur within regions as factor data changes.

11 The effective number of stocks is defined as the reciprocal of the Herfindahl index, which is a commonly used measure of portfolio concentration:

$$\text{Effective number of stocks} = \frac{1}{\sum_{i=1}^{N} w_i^2}$$

where N is the number of constituent stocks in the index and w_i is the weight of stock i in the index. In brief, the effective number of stocks in a portfolio indicates how many stocks would be needed in an equal-weighted portfolio to obtain the same level of concentration (as measured by the Herfindahl index). Equal-weighting stocks in a portfolio will lead to the maximum effective number of stocks.

markets currency risk within the strategy. The currency hedge for investors in other regions will depend on the correlation of the home currency with global equity markets. We believe the currency hedge reduces volatility over the long-term while not sacrificing return (see Joiner and Mollan [2017]).

While there is an initial allocation suggested, we believe it is necessary to have an ongoing monitoring process in place to change elements of the strategy in order to be able to continue to deliver on its objectives. Nevertheless, we would expect any changes, including the regional allocation, to be gradually made over time reflecting the strategic rather than tactical nature of the process.

Conclusion

We have demonstrated above the evidence and beliefs that underpin the design choices we make for our multi-factor strategy. Choices were made with regard to the selection of factors, the starting universe, the multi-factor construction approach, factor weights and the stock weighting scheme as well as the regional allocation and currency exposures. Explicit consideration was given to many different elements that can influence outcomes.

This level of understanding also allows us to create well-informed objectives for return and risk that can be messaged to investors. For our strategy, based on its exposure to the targeted factors, we aim to, over the long term, outperform a blend of market-cap indices with a similar regional weighting to ours, at a lower level of volatility.

By seeking diversification at multiple layers including at factor, region, sector and stock level, we have designed a solution which we believe will meet the objectives of many investors who are looking for a strategic, long-only exposure to equity factors delivered in a diversified manner.

The views expressed in this article are those of the authors and do not necessarily represent those of their firm.

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Appendix: Geometric S-score and arithmetic S-score

In order to calculate S-scores for stocks, first we calculate z-scores.

$F_{j,i}$ is stock i's attribute value for factor j
μ_j is the cross-sectional mean (ie, the average across all stocks) for factor j
σ_j is the cross-sectional standard deviation (ie, the average across all stocks) for factor j
$z_{j,i}$ is stock i's z-score for factor j

$$z_{j,i} = \frac{\left(F_{j,i} - \mu_j\right)}{\sigma_j}$$

We then apply a winsoring process to the z-scores such that values above 3 are set to 3 and that values below −3 are set to −3. The z-score formula above is re-run with the new values and the winsoring process is applied repeatedly until all z-scores in the universe fall between −3 and 3. We use these winsorised z-scores to calculate the S-scores.

$S_{j,i}$ is stock i's S-score for factor j

$$S_{j,i} = \int_{-\infty}^{z_{j,i}} \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

The winsorised z-score is mapped to an S-score using the cumulative distribution function of the standard normal such that it lies between 0 and 1.

For stock i its geometric S-score (GS) multi-factor score (MFS) across k factors is:

$$GS\ MFS_i = \left(\prod_{j=1}^{k} S_{j,i}\right)^{1/k}$$

And its arithmetic S-score (AS) multi-factor score across k factors is:

$$AS\ MFS_i = \frac{1}{k}\sum_{j=1}^{k} S_{j,i}$$

Average factor rank

For a universe with n stocks, the attribute rank for stock i on factor j is defined by:

$$R_{j,i} = \frac{Rank\left(F_{j,i}\right)}{n}$$

Then the average factor rank (AFR) multi-factor score for stock i with k factors is simply given by:

$$AFR\ MFS_i = \frac{1}{k}\sum_{j=1}^{k} R_{j,i}$$

Smart beta and beyond:
Maximising the benefits of factor investing

Lionel Martellini, Professor of Finance, EDHEC Business School, Director, EDHEC-Risk Institute; **Vincent Milhau**, Research Director, EDHEC-Risk Institute

This article provides clarification with respect to the various possible definitions of factors that are relevant in investment practice.

We develop a framework for allocating to factors in two main contexts, namely allocation decisions at the asset class level, and benchmarking decisions within a given class.

Several definitions for factors co-exist, which differ through their focus on return versus risk, or on cross-sectional differences between assets versus the time-series properties of assets.

The various notions of factors are not mutually exclusive and can be combined within a comprehensive framework for factor allocation.

It is possible to use factor indices as building blocks and to diversify risk across underlying factors, or to seek to exploit knowledge of economic regimes to design portfolios that react to changes in market conditions.

Factor investing is an investment paradigm under which an investor decides how much to allocate to various factors, as opposed to various securities or asset classes. Its popularity has been growing since the turn of the millennium, especially after the recognition in 2008 that multiple asset classes can experience severe losses at the same time despite their apparent differences. The term 'factor', however, is used with many different meanings depending on the context and the targeted application. The main goal of this article is to provide clarification with respect to the various possible definitions of factors that are relevant in investment practice. This article also develops a framework for allocating to factors in two main contexts, namely allocation decisions at the asset class level, and benchmarking decisions within a given class. For each of these applications, we examine the three most important questions raised by the adoption of a factor investing approach: (i) why think in terms of factors? (ii) what factors should be chosen? and (iii) how do we allocate between them?

Several definitions for factors co-exist, which differ through their focus on return versus risk, or on cross-sectional differences between assets versus the time-series properties of assets. A distinction can be made between (i) asset pricing factors, (ii) strategies that deliver a positive premium in the long run, (iii) common sources of risk in various assets and (iv) state variables that characterise current business conditions.

Asset pricing theory is concerned with the search of 'asset pricing factors', defined as factors that explain the cross section of expected returns in the following sense: the expected returns of various assets are completely determined by the exposures of these assets to the factors, the exposures being obtained by running a multivariate regression of asset returns on factor values. The premium of a factor measures the incremental reward received in the form of additional expected return by increasing the exposure. According to the theory, it is driven by the covariance between the factor and the 'marginal utility of consumption' of the representative agent, which is the gain in utility for a small increase in consumption. A factor is positively rewarded if it tends to be high in 'good times', defined as scenarios in which consumption is high (and consequently marginal utility is low) and low in 'bad times', defined as scenarios in which consumption is low. This is because bearing exposure to this factor tends to generate a high payoff when it is least needed as consumption is already high, and a low payoff when additional money is most valued, meaning that this exposure is unattractive unless it is rewarded by higher return in the long run.

In investment practice, the notion of a factor is more polysemic. A factor can be a profitable strategy that delivers a long-term premium over a benchmark, provided this premium is economically justified as a reward for bearing additional risk, like in asset pricing theory, or as the result of biases in investors' behaviour that cannot be completely eliminated due to the existence of limits to arbitrage. This definition applies well to the passive equity strategies that select stocks based on some observable characteristic: low size, high value, high momentum, low volatility, high profitability and low investment are sources of long-term returns documented by extensive academic research and backed by sound economic rationale. Capturing these premia at reasonable cost is the goal of 'equity factor indices' offered by equity index providers. In non-equity classes, research is more recent, so our understanding of risk premia is comparatively more limited. Value, momentum and carry are three effects that have been reported for bonds and commodity and currency futures, but the list is likely still incomplete, and further research is needed to study the existence and the persistence of rewarded factors, in particular in fixed income securities, which is a major asset class for institutional investors.

Another notion of a factor is that of the risk factor, and it refers to common sources of risk that affect various securities or asset classes. Volatilities and correlations are then mainly explained by the exposures to these factors, and common exposures can result in joint losses in severe bear markets, like in 2008. Several macroeconomic variables such as output, growth and inflation can play this role, but in order to maximise the explanatory power, risk factors are often taken to be implicit, that is they are extracted from asset returns by statistical analysis. In the Barra equity model, implicit factors are intended to represent the common sources of risk that affect assets with similar microeconomic characteristics. Specific statistical procedures can also be used to obtain factors with a zero correlation, a property that facilitates the decomposition of the risk of a portfolio. These procedures are named principal component analysis and minimum linear torsion (see Carli, Deguest and Martellini [2014] for a review, and the example of implementation below).

Finally, a third possible definition for a factor in practice is as a state variable that contributes to explaining time variation in the risk premia, volatilities and correlations of assets. This definition takes a time-series perspective, unlike the previous ones, which aim to explain cross-sectional properties. The risk and return characteristics of assets can be compared across regimes defined in terms of macroeconomic variables that have an impact on discount rates or expected future cash flows. It is also standard practice to take state variables as the dividend yield as a predictor of stock returns, or use the forward-spot spread to predict bond returns.

It should be noted that financial theory establishes connections between the three practical categories of factors and the notion of the pricing factor: the risk-based explanation for the profitability of passive strategies is that they are exposed to rewarded pricing factors, the Arbitrage Pricing Theory of Ross shows that common risk factors can be pricing factors, and the Intertemporal Capital Asset Pricing Model of Merton implies that state variables that predict changes in investment opportunities are pricing factors.

At the asset class level, risk factors allow the diversification of a portfolio to be assessed in a more meaningful way than dollar weights, and they are involved in the construction of liability-hedging portfolios by factor-matching techniques. Conditioning factors are useful to design performance-seeking portfolios that react to market conditions.

Modern portfolio theory gives a clear definition of what a 'well-diversified' portfolio should be: it should have the highest Sharpe ratio, equal to the reward, measured as expected excess return over the risk-free rate, per unit of risk, measured as volatility. But this prescription is hard to implement in

1. Diversification of risk factors

(a) Effective number of uncorrelated bets for selected portfolios

	PCA factors	MLT factors
Policy portfolio	1.34	3.40
Equally-weighted	1.08	3.77
Risk parity	2.00	6.00
Minimum variance	2.67	2.28

(b) Composition of risk parity and factor risk parity portfolios (%)

	US equities	International equities	US Treasuries	US corporate	US TIPS	Commodities	Real estate	Total
Risk parity	7.4	6.3	40.6	16.4	18.5	6.2	4.6	100.0
Factor risk parity – PCA	13.7	–11.7	98.9	–33.3	12.0	15.7	4.7	100.0
Factor risk parity – MLT	15.0	4.3	42.1	18.2	15.5	5.3	–0.4	100.0

Note 1: Constituents are US equities, world ex-US equities, US Treasuries, US corporate bonds, US Treasury Inflation-Protected Securities, commodities and US real estate. Composition of policy portfolio is 30% in US equities, 30% in international equities, 15% in US Treasuries, 15% in US credits, 3.33% in TIPS, 3.33% in commodities and 3.33% in real estate. Sample period goes from April 1997 to September 2017.

Note 2: By construction, the factor risk parity portfolio is not unique, so we select the one with the lowest leverage (sum of absolute values of short positions) in Panel (b).

2. Conditional means and volatilities of asset classes in inflation-growth regimes (%)

Average returns

Regime	High growth Low inflation	High growth High inflation	Low growth Low inflation	Low growth High inflation	Unconditional
US equities	23.0	10.7	9.0	5.8	11.1
International equities	23.4	13.9	9.4	–0.4	10.4
US Treasuries	4.4	8.0	6.5	9.5	7.4
US credit Baa	5.7	10.3	10.0	7.4	8.6
Commodities	–3.1	17.6	–2.5	17.0	8.7

Volatilities

Regime	High growth Low inflation	High growth High inflation	Low growth Low inflation	Low growth High inflation	Unconditional
US equities	15.0	14.2	17.0	18.9	16.6
International equities	17.4	15.1	19.6	22.4	19.2
US Treasuries	4.9	5.4	5.6	7.3	6.0
US credit Baa	4.7	7.1	6.5	12.5	8.6
Commodities	20.1	18.5	23.6	26.6	23.0

Sample period is from January 1973 to December 2016.

practice, due to the strong uncertainty over expected return estimates, which research has shown to have a dramatic impact on performance. To alleviate the concern over parameter uncertainty, one may decide to go back to conventional wisdom and diversify by 'spreading eggs across baskets', which hopefully leads to more efficient collection of risk premia across assets. A standard interpretation of this principle is to weight constituents equally, but it opens the door to portfolios with concentrated risk: the risk of a 50%–50% stock-bond portfolio is mostly explained by stocks.

By equating the contributions of assets to risk, the risk parity approach to allocation is a big step towards addressing this issue, but it still misses the fact that constituents are exposed to common sources of risk. To assess the level of diversification of a portfolio in terms of risk factors, we propose to calculate the effective number of uncorrelated bets (ENUB), a quantitative measure of the deconcentration of factor contributions to portfolio volatility that is minimal when risk is concentrated in a single factor, and maximal when all factors contribute equally to risk. The latter condition defines a factor risk parity portfolio.

Factor contributions are easiest to calculate when the factors are uncorrelated from each other because there are then no cross-correlation terms to divide between factors. As introduced earlier, uncorrelated risk factors that completely explain uncertainty in a given universe can be obtained by (at least) two statistical procedures, namely principal component analysis (PCA) and minimum linear torsion (MLT). The latter method was introduced more recently, and it aims to address some of the shortcomings of PCA by minimising the distortion of factors with respect to the original assets: this property facilitates the economic interpretation of factors and enhances robustness across samples.

Figure 1 shows an example of the ENUB calculation in a seven-asset class universe mixing equities, bonds, commodities and real estate. The four benchmark portfolios have ENUBs much lower than the theoretical maximum of seven, which means that their risk is concentrated in a few risk factors, except for the risk parity allocation when MLT factors are employed: indeed, each MLT factor is close to an asset, so the risk parity portfolio should not be exceedingly far from a factor risk parity portfolio. Nevertheless, the true factor risk parity portfolio for MLT factors has a different composition than the risk parity one. With PCA factors, it is virtually impossible to achieve factor risk parity with a long-only allocation, since the first factor will inevitably dominate the others, so sizeable short positions must be taken. This example illustrates the fact that MLT factors are computationally easier to handle.

Risk factors are also naturally involved in a different context, where the objective is not to efficiently diversify across assets, but to replicate a benchmark as closely as possible, like in asset-liability management, where a good liability-hedging portfolio (LHP) is needed. Through the discounting mechanism of future cash flows, interest rate risk is a major source of risk, and often the dominant one, in liabilities, so aligning the interest rate exposures of assets and liabilities is the first step towards the construction of a LHP. The difficulty here is that exposures are not linear, so linear approximations are needed. The first-order approximation leads to duration matching, which is effective at immunising the funding ratio against small changes in the yield curve, but in order to hedge against the effects of larger changes, finer approximation is required, involving a matching of convexities in addition to duration alignment.

Beyond risk factors, state variables characterising time-varying investment opportunities may prove useful in asset allocation, in order to construct a performance-seeking portfolio that adapts to market conditions. A simple way to define regimes is to look at inflation and growth in gross domestic product and to make a distinction between four regimes, depending on whether inflation and growth are below or above their mean values. The results in figure 2 show that equities do best when inflation is modest and growth is dynamic, while the low growth and high inflation regime is the least favourable to them, both in terms of performance and volatility. Commodities perform better in the high inflation than in the low inflation periods, and Treasuries deliver their best performance in the low growth and high inflation regime, thereby confirming their role of a 'safe haven'. These results suggest that regimes of growth and inflation can be used to adapt the relative weighting of asset classes as a function of market conditions.

Within an asset class, theory makes a case for factor investing by showing that the maximum Sharpe ratio (MSR) portfolio of individual securities coincides with the MSR portfolio of pricing factors, no matter how large the original universe is. Empirically, equity factor indices representing the six well-documented factors (size, value, momentum, volatility, profitability and investment) dominate the standard cap-weighted index in terms of risk-adjusted return, especially if they are 'smart weighted'. Further improvement over the risk-return characteristics of individual factors is achieved by building multi-factor portfolios.

Going back to the theoretical definition of a well-diversified portfolio as the MSR portfolio, a theoretical result that we prove in this paper is that the MSR portfolio of any set of assets coincides with the MSR portfolio of factors, provided the latter are pricing factors in the sense of asset pricing theory. This result holds regardless of the number of assets, so it represents a substantial reduction in dimensionality if there are many of them, as is generally the case in benchmarking. It also provides the optimal form of the 'two-step process', in which an allocation exercise to multiple securities is divided into two steps, namely the grouping of securities in benchmarks, and then an allocation to the benchmarks.

This theoretical result cannot be directly applied in practice to calculate the MSR portfolio because a complete set of pricing factors is not known, but the idea of dimension reduction can be exploited with other types of factors, namely risk factors. Indeed, under a factor model, each return can be decomposed into a systematic part that is a sum of factor exposures, plus an idiosyncratic term, and provided idiosyncratic returns are uncorrelated across assets, the number of independent parameters to estimate in the covariance matrix is much smaller than if no factor structure is postulated. Considering for instance a universe of N = 500 stocks, it is shown in the paper that the number of covariances is 125,250 without a factor model, 3,521 with six factors and 2,006 with three of them. In other words, the use of risk factors alleviates the curse of dimensionality for the estimation of the covariance matrix. This idea is implemented in Barra models, as explained in the Barra Risk Model Handbook.

Although a comprehensive set of pricing factors has not been uncovered to date, it is well known from a large body of empirical research that at least in the equity class, factors understood as profitable strategies provide a substantial improvement over the standard cap-weighted index in terms of risk-return characteristics. Figure 3 summarises this evidence by assuming factors to be a set of long-only factor indices made of stocks with a given characteris-

3. Effects of selection and weighting in equity benchmarks

	All stocks			Mid cap			Value			High momentum			Low volatility			High profitability			Low investment		
	CW	EW	IV	CW	EW	IV	CW	EW	IV	CW	EW	IV	CW	EW	IV	CW	EW	IV	CW	EW	IV
Annualied returns (%)	10.80	12.76	12.83	13.42	14.13	14.10	12.29	14.54	14.48	10.80	12.76	12.83	13.42	14.13	14.10	12.29	14.54	14.48	10.80	12.76	12.83
Annualised volaility (%)	6.90	16.76	16.12	17.12	17.06	16.29	17.19	16.72	16.16	16.90	16.76	16.12	17.12	17.06	16.29	17.19	16.72	16.16	16.90	16.76	16.12
Sharpe ratio	0.35	0.47	0.49	0.50	0.54	0.56	0.43	0.58	0.59	0.35	0.47	0.49	0.50	0.54	0.56	0.43	0.58	0.59	0.35	0.47	0.49

Index values are from the ERI Scientific Beta database and span the period from June 1970 to December 2015. CW means 'cap-weighted', EW stands for 'equally-weighted' and IV for 'inverse volatility'.

4. Properties of multi-factor equity portfolios

	Annualised return (%)	Annualised volatility (%)	Sharpe ratio	Maximum drawdown (%)	Tracking error (%)	Information ratio	Maximum relative drawdown (%)
Constituents' average	11.62	16.88	0.40	53.74	4.19	0.29	25.92
Equal weight	11.74	16.45	0.41	52.30	1.97	0.72	13.32
Minimum variance (long-only)	11.25	15.46	0.41	47.84	3.98	0.23	36.90
Risk parity	11.76	16.36	0.42	51.96	2.09	0.69	14.73
Maximum ENUB (long-only)	11.53	15.56	0.42	48.30	3.31	0.37	30.80
Maximum relative ENUB (long-only)	11.09	16.81	0.37	53.18	1.12	0.69	5.18

The sample period is from July 1972 to December 2015, and portfolios are rebalanced quarterly. Minimum linear torsion, and statistics are computed out of sample.

tic: mid market capitalisation, high book-to-market, high past one-year return, low volatility, high gross profit-to-asset ratio or low total asset growth. The base case version of these indices is cap-weighted, but 'smart factor indices' deviate from this weighting scheme in order to better diversify away unrewarded risk. Among the many possible schemes, the table considers equal weighting and inverse volatility weighting. Over the long period considered, all factor indices outperform the broad cap-weighted index and display a higher Sharpe ratio, and the smart versions bring further improvement on these figures.

The equivalence result between the MSR portfolio of securities and the MSR portfolio of pricing factors suggests that it is interesting to combine factors. Thus, the next exercise that we conduct consists in the construction of multi-factor equity portfolios. Figure 4 shows statistics for selected allocations. Given that the six factors are long-only, they are all exposed to the market equity factor and they have high correlations, greater than 90%, so one may wonder what benefits can be expected from mixing such highly correlated constituents. It turns out that the annualised long-term return and the Sharpe ratio are only marginally improved with respect to the average properties of the constituents, but the relative analytics, which measure risk and return with respect to the broad cap-weighted index, are much more favourably impacted. This can be attributed to the fact that the relative correlations, that is the correlations between excess returns, are much lower than the absolute correlations and are often negative, so diversification can be expected to be more effective from a relative perspective. In particular, the equally-weighted portfolio has much lower tracking error and a maximum relative drawdown, as well as a much higher information ratio than the average of the constituents.

The choice of the allocation method has important effects on the properties of the multi-factor portfolios.

The global minimum variance portfolio achieves its objective even on an out-of-sample basis, but does so at the cost of sizeable relative risk, while the equally-weighted portfolio displays a higher volatility, but better relative analytics. We also calculate two portfolios that maximise diversification in terms of risk factors (subject to a long-only constraint). At this stage, two notions of factors are involved: on the one hand, constituents are profitable passive equity strategies, and on the other hand, the weighting scheme seeks to maximise diversification in terms of underlying risk factors. The latter factors are extracted successively from the covariance matrix of the constituents and from their relative covariance matrix, which collects the covariances of excess returns. Each system of factors gives rise to its own value for the ENUB, and the two ENUBs respectively measure the deconcentration of the volatility and the tracking error. The maximum relative ENUB portfolio has lower relative risk, measured either through the tracking error or the relative drawdown, than its absolute counterpart. Additional backtests the results of which are not reported here but can be found in the complete version of this article show that this finding is robust to the choice of the sample period.

As a conclusion, the various notions of factors are not mutually exclusive and can be combined within a comprehensive framework for factor allocation. Further research is needed to improve our understanding of their interactions, especially in the fixed-income class.

As we argue in the previous empirical illustration, a factor allocation exercise can involve more than one notion of factors. It is possible to use factor indices as building blocks and to diversify risk across underlying factors, or to seek to exploit knowledge of economic regimes to design portfolios that react to changes in market conditions. After five decades of research on equities, robust sources of profitability are now well identified in this class, but not as well in other classes, especially in fixed-income. Moreover, while past research has mostly focused on finding predictors for the equity market or the bond market as a whole, and while it is recognised that factor indices have cyclical behaviour, further investigation is needed to quantify the degree of predictability in these factors and to identify relevant predictors.

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Goal-based investing and its application to the retirement problem

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Goal-based investing principles can be used to effectively address the retirement investing problem by allowing investors in transition to secure minimum levels of replacement income for a fixed period of time in retirement, and also generate the kind of upside needed to reach target levels of replacement income with attractive probabilities.

The emergence of the goal-based investing paradigm has effectively allowed for the development of mass-customised investment solutions to individuals.

Risk management will play a central role in what should be regarded as nothing short of an industrial revolution that is impacting the investment management industry.

The need for new retirement investment solutions

Financing consumption in retirement has arguably become the greatest challenge for most individuals following a number of important changes, including the weakening state pension systems and the shift from defined-benefit to defined-contribution schemes in the corporate world that has left individuals more exposed to retirement risks. With the need to supplement retirement savings via voluntary contributions, individuals are increasingly responsible for their own savings and investment decisions. This global trend poses substantial challenges as individual investors not only suffer from behavioural limitations, but also typically lack the expertise needed to make educated investment decisions.

In response to these concerns, insurance companies, investment banks and asset management firms have proposed a number of so-called retirement products. There are reasons to believe, however, that these products fall short of providing satisfactory solutions to the problems faced by individuals when approaching investment saving decisions. In this paper, we describe how goal-based investing principles can be used to design scalable mass-customised forms of retirement solutions that can address the specific retirement needs and constraints of a large number of individuals in a parsimonious manner. As an example of the framework in application, we propose a goal-based investing strategy for retirement needs in accumulation that can be regarded as a simple and pragmatic risk-managed improvement over existing forms of target-date funds, making them better suited to investors who are saving for retirement in the accumulation phase of their life cycle. In parallel, and in an effort to help increase awareness around the need for improved retirement solutions, EDHEC-Risk Institute and the Princeton Operations Research and Financial Engineering (ORFE) Department have teamed up to launch the EDHEC-Princeton Goal-Based Investing index series. These indices are based on joint academic research conducted with the support of Merrill Lynch Wealth Management on the application of goal-based investing (GBI) principles to the retirement problem.[1]

A careful analysis of retirement investment solutions is rather timely – on 29 June 2017, the European Commission published a legislative proposal for a regulation on a pan-European personal pension product (PEPP). According to the proposal, PEPP providers shall offer up to five investment options to PEPP savers, including a default investment option. In its current format, the Commission's text (article 37.2) suggests that the default option could be accompanied by a guarantee. While it seems intuitively desirable that the default option should aim to preserve capital over time, one key concern is that the introduction of minimum return or capital guarantees would have a number of negative consequences. The most important of these consequences would be an exceedingly large opportunity cost for beneficiaries, given the presence of strict prudential regulations such as Solvency II, which make such guarantees prohibitively expensive.

In addition to the direct opportunity cost deriving from the introduction of a formal insurance guarantee, as well as the costs implied by the typical distribution channels for such guaranteed products, one may also be concerned by the indirect opportunity costs implied by the use of low-yielding fixed-income instruments in the hedging component of the guaranteed products. Moreover, the typical use of single-class liquid underlying instruments such as stock indices for guaranteed products (as opposed to well-diversified multi-asset portfolios) may also contribute to a lack of diversification.

In this context, the enhanced upside potential offered by life-cycle strategies, also known as target-date fund strategies, may seemingly make them attractive alternatives due to the fact that these are inherently designed as long-horizon strategies that explicitly benefit from the well-documented presence of mean-reversion in risk premia to be found in the equity market and beyond.

On the other hand, target-date funds offer a sole focus on an investment horizon without any protection of investors' minimum retirement needs. In particular, these products are not engineered to deliver replacement income in retirement, and do not adequately hedge the main risks related to retirement investing decisions, namely investment risk, interest rate risk, inflation risk and longevity risk. Another important restriction is that most target-date funds do not allow for revisions of the asset allocation as a function of changes in market conditions. This is entirely inconsistent with academic prescriptions and also, perhaps more importantly, with common sense, which both suggest that a meaningful investment strategy should also display an element of dependence on the state of the economy as well as a dependence on investors' goals.

Replacement income, not absolute wealth, should be the focus!

Currently available investment options hardly provide a satisfying answer to the retirement investment challenge and most individuals are left with an unsatisfying choice. On the one hand, they have safe strategies with very limited upside potential, which will not allow them to generate the kind of target replacement income they need in retirement; on the other hand, they have risky strategies offering no security with respect to minimum levels of replacement income.

The most natural way to frame an investor's retirement goal is in terms of how much lifetime guaranteed replacement income they will be able to afford at retirement. More often than not, investors in accumulation are concerned with the purchasing power of their replacement income in terms of consump-

1 The launch is scheduled to take place in early Q2 2018, and the performance of the EDHEC-Princeton GBI indices will be posted on both the EDHEC-Risk Institute and Princeton ORFE websites..

tion goods and services in retirement. Given that the biggest risk in retirement is the risk of outliving one's retirement assets, securing replacement income within the decumulation period can be achieved with annuities (possibly inflation-linked or cost-of-living-adjusted), which are the true risk-free assets for individuals preparing for retirement. Annuity products, however, are cost-inefficient, irreversible and do not contribute to bequest objectives. These elements undoubtedly explain the low demand for annuities, aka the 'annuity puzzle', that is of course when annuitisation is not incentivised or mandatory. A good case can actually be made that annuitisation is a decision that is best taken close to retirement, if ever.

In the UK, the 2015 Pension Act, which has nullified the compulsory annuity purchase, creates a tremendous opportunity for asset managers to launch meaningful forms of retirement solutions. A key ingredient in these retirement solutions is a novel form of retirement bond portfolio, where the key focus should be on generating replacement income for a period roughly corresponding to the average life expectancy in retirement (say for 15 or 20 years after the retirement date).

In parallel, late-life annuities can be purchased in decumulation to obtain protection against tail longevity risk. It would actually be extremely useful for governments and central banks to start issuing these 'retirement bonds'.[2] While most existing bonds are useful for corporations and sovereign states to finance their activities, they are not useful investment vehicles for investors. Indeed, investors in the accumulation phase of their life cycle do not need a stream of coupon payments plus principal at maturity date, which is the typical structure of available bond offerings. What individuals need are bonds paying no cash flow in the accumulation phase, that is no cash flows until the retirement date, and then paying monthly, quarterly or annual (possibly inflation-linked) coupons for a given number of years (eg, 15 or 20 years in retirement) and no principal at the maturity date.

In the absence of such retirement bonds, forward-start bond ladder structures can be synthesised via standard cash flow-matching or duration-matching techniques to obtain a dedicated retirement goal-hedging portfolio (GHP). Purchasing $1 worth of face value of the synthetic retirement bond is thus equivalent to securing an additional $1 worth of replacement income (possibly inflation-linked) say for the first 15 or 20 years in retirement.

To illustrate the fact that assets such as a Treasury bond portfolio or a money market account (which are traditionally regarded as safe investments) are actually highly risky when it comes to securing a stream of replacement income cash-flows, figure 1 plots the monthly returns on these investments in absolute terms and relative to the present value of replacement income. Returns on money market accounts (cash) are very stable and consistently close to zero, while Treasury bond returns exhibit more short-term volatility. Note that they both appear much less volatile than the returns on the GHP, which is more exposed to interest rate risk because of its long duration. Note, however, the picture is completely different when returns are computed with respect to the retirement bond price (ie, relative to purchasing owner in terms of replacement income). By construction, the GHP does indeed have zero relative risk, while cash and bonds now appear to be highly risky. Overall, the distinction between absolute and relative risk, which is well established in asset-liability management, is also of key relevance in the retirement funding problem: replacement income, not absolute wealth, should be the focus!

Given the price of the retirement bond (ie, given the market value of replacement income cash flows), it is straightforward to calculate the purchasing power of a given level of retirement savings in terms of replacement income (ie, the level of replacement income that these savings can finance). It is equal to the value of savings divided by the retirement bond price. As such, the retirement bond price, which provides the proper reference point, or numeraire, is an important piece of information in goal-based reporting. In what follows, we argue from a risk management standpoint that it is also useful for the construction of strategies that maximise the probability of reaching target levels of replacement income.

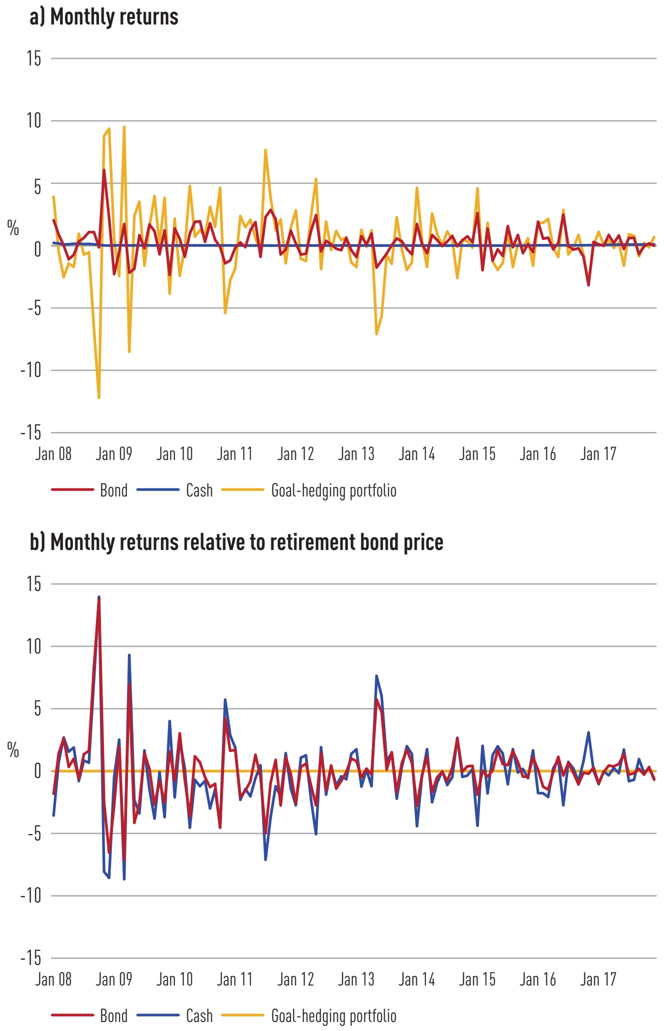
Key requirements for improved goal-based retirement solutions

Individuals can set target levels of replacement income expected from retirement savings as a function of their estimated consumption needs in retirement as well as income generated by other sources such as Social Security and employer-sponsored pension plans. Should a replacement income target be affordable given the current level of retirement, it could be secured by investing the required amount of wealth in the GHP.

In most cases, however, individuals and households are under-funded: their replacement income needs in retirement exceed what can be financed via savings alone. In other words, the desired replacement income level is not affordable and therefore represents an aspirational goal (in the terminology of

2 A similar argument was put forward by Robert C Merton and Arun Muralidhar in an article entitled, Time for retirement 'SeLFIES'?', published in the April 2017 issue of IPE.

1. Absolute risk versus relative risk



The retirement bond is for an investor who plans to retire in January 2018, has a 15-year decumulation period and targets a constant replacement income (no inflation indexation or cost of living adjustment is required). The bond is represented by the Barclays Treasury index, the cash account earns the secondary market rate on US three-month Treasury bills, and the goal-hedging portfolio replicates the retirement bond price.

Chhabra et al [2015]), the presence of which justifies the need for upside performance. In this context, a well-designed retirement solution should simultaneously generate a high probability for individuals to achieve their aspirational/target levels of replacement income, but it should also secure some essential/minimum levels of replacement income in order to ensure that basic needs in retirement will be satisfied regardless of market performance.

The recognition that investors aspire to secure both essential and aspirational goals with high probabilities is leading to the new GBI investment paradigm in individual money management, where investors' problems can be fully characterised in terms of their goals. Goal-based investing is the counterpart of liability-driven investing (LDI), which has become the relevant paradigm in institutional money management where investors' problems are broadly summarised in terms of their liabilities.

From a financial engineering standpoint, any GBI retirement solutions should be grounded on sound and robust risk-management principles and involve the following ingredients:

- A dedicated safe GHP that replicates risk factor exposures in investors' replacement income goals (dynamic replicating bond portfolio for the aforementioned retirement bonds);
- A common well-rewarded risky performance-seeking portfolio (PSP) that efficiently harvests risk premia in equity markets; and
- A dynamic allocation to the PSP versus GHP portfolios that secures minimum replacement income levels while generating a high probability of achieving target replacement income levels.

As such, the framework builds upon a comprehensive and holistic integration of the three forms of risk management, namely hedging, diversification, insurance, in contrast with existing products or approaches used in institutional or individual money management, which are only based on selected risk management principles. While each of these sources of added value is already used to some extent in different contexts, a comprehensive integration of all these elements within a comprehensive disciplined investment management framework is required for the design of useful investment solutions. In the next section, we provide an example of implementation of goal-based investing principles applied to retirement, and present design features that have been used in the EDHEC-Princeton Goal Based Investing index series.[3]

Introducing a new generation of risk-managed target-date retirement solutions

Let us consider for concreteness an investor preparing for retirement who seeks to obtain protection on a yearly basis with respect to the purchasing power in terms of replacement income in decumulation of any contribution made in accumulation or transition phases. Assuming for simplicity that contributions are made once a year, say at the end of December, one would naturally introduce the essential goal to cap the loss relative to replacement income to a fixed limit – eg, 20%, over a calendar year. This short-term essential goal commands a floor that the strategy should respect at all times, and is equal to 20% of the price of the retirement bond that pays the replacement income that was affordable at the beginning of the year. This floor is reset every year to be equal to 80% of current savings, including the annual contribution.

This mechanism is depicted in figure 2, where we plot the value of accumulated savings and the level of affordable income for an investor who starts with $10,000 in January 2010 and adds another $10,000 every year to his/her account. The floor expressed in terms of affordable income is by definition equal to 80% of the income level that was affordable in January, so it is constant within a year.

Protection of the floor can be achieved by the means of a dynamic insurance strategy, in which the dollar allocation to the PSP is taken to be a multiple of the risk budget or margin for error, defined as the distance between current wealth and floor levels. Thus, if $w_{PSP,t}$ denotes the percentage allocation to the PSP and m_t is the (time-varying) multiplier, we obtain an allocation that reacts to changes in the risk budget according to the following linear rule, with a rebalancing frequency taken to be monthly in our base case analysis:[4]

$$w_{PSP,t} = m_t\left[1 - \frac{F_t}{W_t}\right]$$

In order to anchor the design of the retirement GBI solutions with respect to existing target-date fund, we set the value of the multiplier at the beginning of every year in such a way that the percentage allocation to the PSP, taken for simplicity to be some equity index, matches the equity allocation of a deterministic target-date fund. This allows us to benefit from mean-reversion in equity markets, which implies that the allocation to equities should be higher for younger investors. With this rule, the multiplier is the deterministic function plotted in figure 3, and the GBI strategy has exactly the same allocation to its performance-seeking equity component as the corresponding target-date fund at the beginning of each year. Within any given year, however, the allocation to equities does not stay constant and instead reacts to changes in the distance between current wealth and the floor, to protect the essential goal.[5]

To compare the risk-managed target-date retirement strategy to its standard target-date fund benchmark, we simulate 10,000 scenarios for equity returns and interest rates, and we look into the evolution of the level of affordable income over the accumulation phase. As argued before, this indicator is more relevant than the absolute performance of the strategy in the retirement financing context. Formally, we calculate a 'funding ratio', defined here as the ratio of the current level of affordable income to the initial level of affordable income.

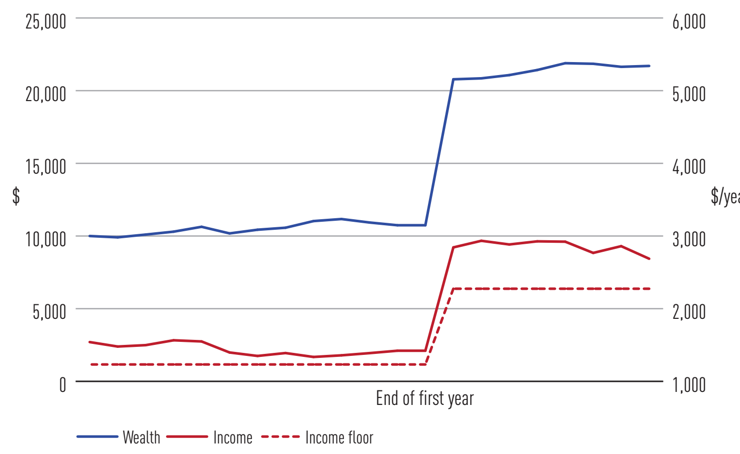
This quantity is independent from the capital invested in the strategy and it measures the performance of the strategy relative to the retirement bond price. It would be constant at 100% for a portfolio fully invested in the GHP, and it grows above 100% if affordable income increases. In order to isolate the effect of the investment strategy, we assume in these simulations that no further contributions take place after inception.

Figure 4 reports a series of ex-ante indicators on the distribution of future funding ratios. To obtain these numbers, assumptions must be made on the dynamics of returns and risk factors impacting prices. We simulate the returns on an equity index by setting its annual volatility to 16.2% and its Sharpe ratio to 0.395, two values that are consistent with long-term risk and return estimates for the S&P 500 index. The bond component of the target-date fund is modelled as a portfolio with 6.4% volatility and 0.234 Sharpe ratio, and the GHP of the risk-controlled strategy replicates the returns of the retirement bond for an individual who retires in January 2038. This retirement bond is priced as the discounted value of future cash flows given the current term structure of interest rates.

For parsimony, we assume a one-factor interest rate model, the parameters of which are calibrated to historical series of US zero-coupon rates spanning

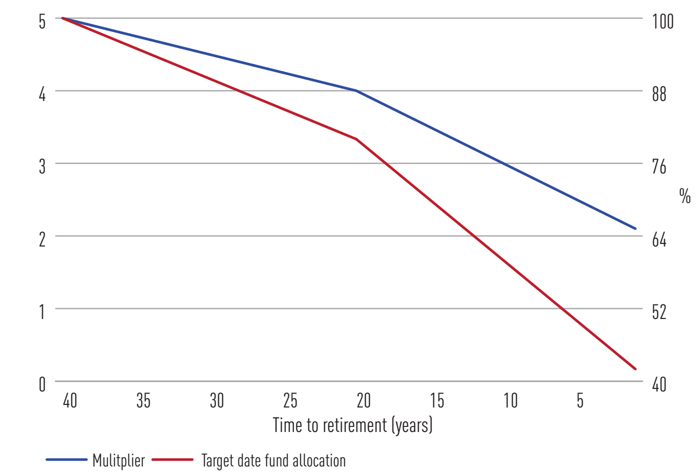
3 For more detail, see Giron et al (2018).
4 The allocation to the PSP is typically capped to 100% to avoid leverage.
5 In implementation, it would also be useful to make it a function of market conditions, based on the finding that higher volatilities and lower expected returns should imply lower multiplier values, and that conversely, lower volatility and higher expected returns should result in higher multiplier values.

2. Floor reset



A contribution of $10,000 is made at date 0 and then at the end of every year during the accumulation phase.

3. Multiplier glide path



4. Ex-ante reporting for goal-based investing retirement strategies and target-date funds

	Target-date fund	GBI strategy	GBI strategy with improved PSP
Expected funding ratio at retirement (%)	207.7	206.0	319.7
Probabilities of reaching funding rates of ... (%)			
130%	88.4	85.9	96.1
150%	78.9	75.4	92.2
200%	52.9	50.7	78.1
Annual volatility (%)	10.7	12.4	13.3
Probabilities of annual losses greater than ... (%)			
10%	16.1	84.3	72.0
20%	84.6	0.0	0.0
Worst loss (%)	35.7	18.7	18.5

These numbers are obtained by simulating 10,000 scenarios for the target-date fund, the GBI strategy and the retirement bond price for an individual who starts accumulating in January 2018 and expects to retire in January 2038. The improved PSP is simulated by raising the Sharpe ratio of the base case PSP by 50%. Rebalancing is assumed to take place at a monthly frequency.

the period from January 1998 to January 2018.[6] With the estimated parameters, the GHP has a volatility of 5.4% on average (decreasing over time as duration decreases) and a mean return of 3.05%. We emphasise that these parameter values are only needed to simulate future scenarios, but that they are not involved in the implementation of the GBI strategy.

When analysing the results displayed in figure 4, it appears that risk-managed target-date GBI retirement solutions are comparable to conventional target-date funds in terms of long-term expected funding ratio and probabilities of reaching aspirational levels of funding. On the other hand, standard forms of target-date funds are unable to reliably secure annual losses to the specified level of 20%, with a 16.1% probability of experiencing at least one loss above this threshold over the period, when the GBI strategy reaches the objective of securing 80% of the initial annual funding ratio in all scenarios.[7] In the most extreme negative scenario in our simulations, the worst loss in terms of funding ratio for the target-date fund exceeds 35%, while it does not exceed the 20% limit set as an essential goal for the GBI strategy. Interestingly, realistic improvements to the PSP, which can be obtained by shifting from a cap-weighted index to a well-diversified portfolio of smart factor indices, would lead to extremely significant increase in the probability for investors to achieve their target levels of replacement income. For example a 200% increase in purchasing power can be obtained with close to 80% probability (78.1% given our parametric assumptions) for the GBI strategy with an improved PSP, to be compared with about 50% probability for both the target-date fund and the GBI strategy with a poorly diversified cap-weighted equity portfolio.

6 Details on the calibration procedure can be found in Martellini and Milhau (2017).

7 In robustness checks, we have found that some gap risk arises when the GBI strategy is rebalanced quarterly, as opposed to monthly. On the other hand, gap risk is limited in probability (0.2%) and in severity (worst annual loss at 23.4%).

Mass customisation in retirement investing

Goal-based investing principles can be used to effectively address the retirement investing problem by allowing investors in transition (say from age 55 to 65) to secure minimum levels of replacement income for a fixed period of time (say 15 years) in retirement, and also generate the kind of upside needed to reach target levels of replacement income with attractive probabilities. At retirement date (say at age 65), an investor may decide on how to split the available surplus in two components, one dedicated to securing more replacement income for the early stage of decumulation and one dedicated to purchasing deferred inflation-linked late life annuities to take care of tail longevity risk above and beyond for the late stage of retirement.

It is only recently that the emergence of the goal-based investing paradigm has effectively allowed for the development of such mass-customised investment solutions to individuals (see Martellini and Milhau [2017] for a detailed analysis). Mass-customisation is facilitated by the convergence of powerful forces. On the one hand production costs are strongly reduced, due to the emergence of smart factor indices as cost-efficient alternatives to active managers for risk premia harvesting. On the other hand, distribution costs are also bound to go down as the trend towards disintermediation is accelerating through the development of fintech and robo-advisor initiatives.

Risk management, defined as the ability for investors, or asset and wealth managers acting on their behalf, to efficiently spend their dollar and risk budgets so as to enhance the probability to reach their meaningful goals, will play a central role in what should be regarded as nothing short of an industrial revolution that is impacting the investment management industry.

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Predicting risk premia for Treasury bonds:

The ERI Risk Premium Monitor

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Being able to estimate the risk premium attached to Treasury bond yields in a reliable and robust manner is key to successful investing.

It is for this reason that EDHEC-Risk Institute is launching the ERI Risk Premium Monitor: a robust tool to derive a state-of-the-art estimation of the risk premium using market and monetary-policy information.

This article explains how this task is achieved and the theoretical underpinnings of the analytical tools used for the task.

Why risk premia matter

Investors in the Treasury market often observe an upward-sloping yield curve.[1] This means that, by assuming 'duration risk', they can very often invest at a higher yield than their funding cost. Yet, if the steepness of the yield curve purely reflected expectations of future rising rates no money could on average be made from this strategy. This prompts the obvious question: When does the steepness of the yield curve simply reflect expectations of rising rates, and when does it embed a substantial risk premium?

The investment relevance of being able to answer these questions is clear. Take, for instance, a bond manager whose performance is assessed against a Treasury benchmark. Her main strategic investment choices boil down to deciding whether to be long or short duration with respect to the benchmark. Knowing how well she is compensated for taking this duration risk is key to her long-term performance. Or take a multi-asset portfolio manager. Deciding the relative portfolio weights among the different risk factors hinges in great part on the time-varying compensation attaching to these different factors.

In all these cases, and in many more, being able to estimate in a reliable and robust manner the risk premium attaching to yields is key to successful investing. It is for this reason that the EDEHC Risk Institute is launching the ERI Risk Premium Monitor: a robust tool to extract from market and monetary-policy information a state-of the art-estimate of the risk premium.

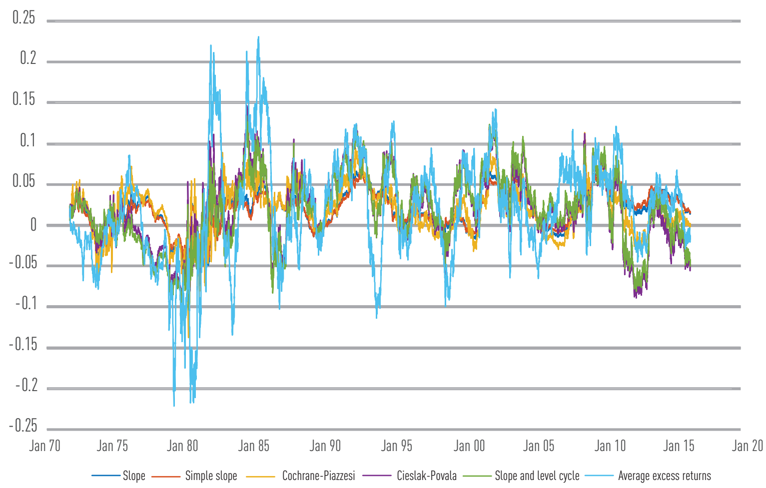
1 Since 1971, the yield curve has been upward sloping (with the 10-year yield above the one-year yield) for almost 84% of the time. Unless investors repeatedly and erroneously expected rates to rise almost all of the time, this is prima facie evidence of the existence of a risk premium.

1. Sharpe ratios

	3-year	5-year	10-year
Full sample	0.20	0.20	0.16
1955–86	0.04	−0.01	−0.07
1987–2014	0.59	0.56	0.49
Recession	0.82	0.72	0.59
Expansion	0.01	0.06	0.05
First-half expansion	0.52	0.50	0.45
Second-half expansion	−0.61	-0.50	-0.48
Tightening cycles			
1979:03–1981:02	−1.06	−1.13	−1.23
1993:03–1985:01	−0.79	−0.86	−0.86
2004:02–2006:02	−1.52	−0.90	−0.50

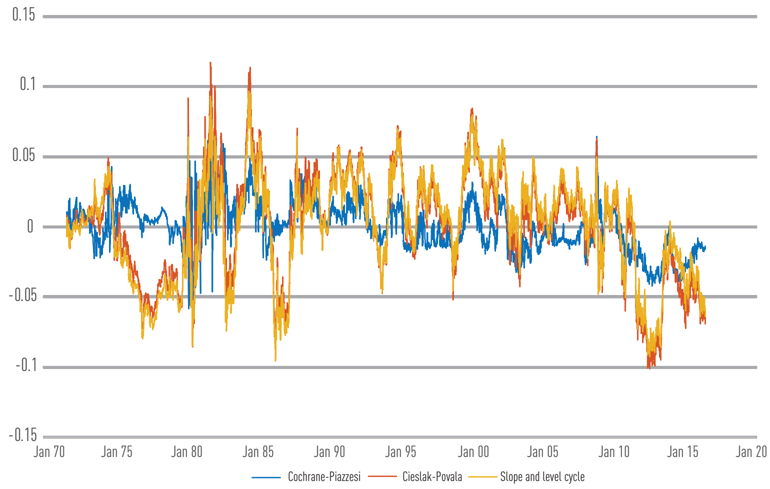
Sharpe ratios for the excess return 'carry' strategy applied to US Treasuries during the 1955–2014 period, subdivided i) into different chronological sub-periods, ii) into periods of recessions or expansions, and iii) during tightening cycles. Data adapted from Naik at al (2016).

2. Average excess returns and various return-predicting factors



Average excess returns from the invest-long/fund-short strategy described in the text, and the excess returns predicted by the slope and by 'new-generation' return-predicting factors (in-sample analysis, US data).

3. Difference between slope and other return-predicting factors



The differences between the prediction of average excess returns produced by the slope, and the predictions produced by the new-generation factors (in-sample analysis, US data).

The rest of this article explains how this task is achieved, and the theoretical underpinnings of the analytical tools used for the task.

Predicting excess returns

What predicts excess returns in Treasury bonds? And how much can one explain? Until recently, the answers to both questions used to be: 'the slope', and 'rather little', respectively. States of the world characterised by a steep upward sloping yield curve used to be considered indicators of positive expected excess returns. The degree of predictability was however modest (with R^2 of the regression of the predicted and realised excess returns never exceeding 20%). To understand why the slope was deemed to be a good predictor of excess returns consider figure 1.

Now, recessionary periods are associated with the monetary authorities cutting rates and therefore engineering an upward-sloping yield curve. It is also natural to assume that in the troubled recession periods investors should become more risk averse. It is therefore plausible to deduce that the yield-curve slope should explains excess returns (see, eg, Fama [1986], Stambaugh [1988], Fama and French [1989], Dahlquist and Hasseltoft [2016]).

Starting from the mid-2000, several results have questioned this received wisdom[2]: these more recent investigations suggest that different return-predicting factors may be far more complex than the simple slope[3]; and their predictions of excess returns sometimes produce much higher R^2. Why is this the case? And what is the economic significance of the new, more complex, factors?

The motivation of the question can be readily understood by looking at figures 2 and 3, which focus on the predictions made by the old- and new-generation factors. More precisely, figure 2 shows the realised average excess returns, and the excess returns predicted by the slope and other 'new-generation' return-predicting factors. While these predictions are all strongly correlated it is clear that the new-generation factors add a substantial twist to the slope story.

Figure 3 makes this intuition clearer by showing the differences between the prediction produced by the slope, and the predictions produced by the new-generation factors (in-sample analysis). Despite the fact that the new return-predicting factors are constructed following very different prescriptions, what is added on top of the slope predictions is reamrkably similar.

This qualitative analysis therefore prompts the following questions:

- Are these 'extra predictions' informative, or, as Bauer and Hamilton (2015) argue, are they just a result of over-fitting?
- Why do such apparently different return-predicting factors produce such similar incremental predictions (with respect to the slope predictions)?
- What is their financial and economic interpretation?

A full answer would take too long a detour (see, eg, Rebonato [2018]). We can however summarise the main findings as follows.

The first insight is linked to the power spectrum of excess returns: one can clearly see both low-frequency (business-cycle) components (well captured by the 'old' slope factor), but also a much higher frequency contribution, that requires higher principal components to be captured. It is (in part) because of its ability to capture these high-frequency components that a factor such as the Cochrane-Piazzesi fares better than the slope by itself.

This is shown in figures 4 and 5. The first figure shows that at all investment horizons there are important contributions from both low- ('business-cycle') and high-frequency components.

When we look at figures 5 (a) and (b), which show the frequency spectrum of the slope factor and of a 'new-generation' factor, we note how the slope recovers the low frequency peaks of the excess returns, but completely misses the medium- and high-frequency components. Contrast this with the power spectrum of the five-year excess returns and one of modern factors, which displays a remarkable match across all frequencies.

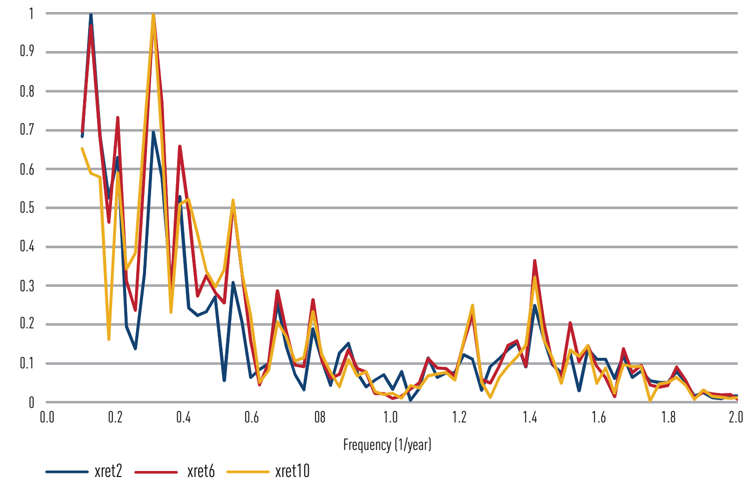
The second 'modern' insight alluded to above suggests that a large fraction of return predictability comes from detecting the cyclical straying of yields from a long-term fundamental trend. Once an effective decomposition of the yield dynamics into trend and cycle is carried out, one finds that the different degrees of mean reversion of the various return-predicting factors explain very well the different degrees of excess returns predictability.

Why do both of these two different 'types of' factors help the prediction of excess returns? We propose that two distinct financial mechanisms can explain excess returns: the first – ie, the one associated with low-frequency changes in excess returns – is linked with changes in risk aversion with business-cycle periodicity. As for the second financial mechanism, associated with higher-frequency cycles, we suggest that it is comes from the actions of pseudo-arbitrageurs who bring back in line with fundamentals the level and slope of the yield curve. These deviations have a much quicker mean-rever-

2 Some reference papers for the new wave of excess-return studies are Cochrane and Piazzesi (2005), Cieslak and Povala (2010a, b), Hellerstein (2011), Rebonato (2015), Dai, Singleton and Yang (2004), and Cochrane (2015).

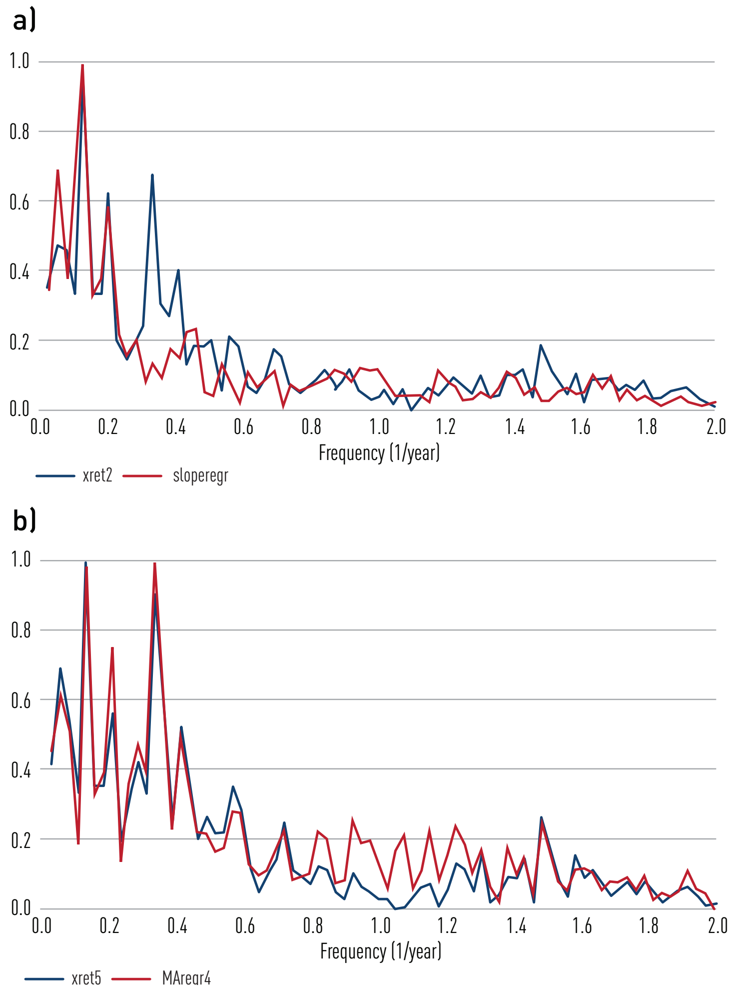
3 For instance, the return-predicting factor of Cochrane-Piazzesi (2005) is usually referred to as a 'tent', and is built by giving weights of different sign and magnitude to five forward rates. In general, the common feature of the new-generation factors is that they require (implicitly or explicitly) much hihger principal components than the second – sometimes as high as the fifth.

4. Frequency components for different excess returns



Normalized frequency components of the power spectrum for the two-, six- and nine-year returns. Frequencies in years⁻¹.

5. Frequency power spectrum

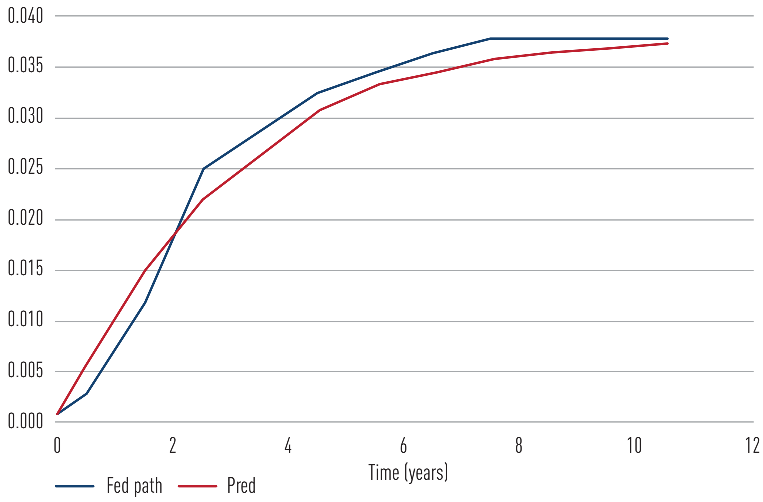


a) Frequency power spectrum of the two-year excess returns and of the slope return-predicting factor.
b) Frequency power spectrum of the five-year excess returns and of the return-predicting factor built using the slope and the cycle to the four-year moving average of the level of yields.

sion, and are therefore associated with the higher-frequency components of the excess return spectrum.

The full picture is more complex, but the key two insights are that the frequency components of excess returns and their mean reverting properties give us a very effective procedure to construct powerful and very parsimonious return-predicting factors: in order to predict excess returns we need a good frequency match (across high and low frequencies), and a good match of the speed of mean reversion. When these two conditions are satisfied, a number of similarly (and highly) effective and robust factors can be built almost by inspection. The new factors are parsimonious (they only require

6. Fed-projected and fitted path of the short rate



This illustrates (i) the Fed path (blue line) and (ii) the most similar path (red line) for the expectation of the P-measure path of the Fed funds.

one slope-like component and one cycle-like component), intuitively understandable (thanks to the financial interpretation offered above) and highly effective (both in-sample and out-of-sample they predict as well as, and often better than, the Cochrane-Piazzesi or the Cieslak-Povala factors).

Regularising the statistical information

Interesting as these results are, all predictions about risk premia gleaned from purely statistical studies suffer from two main shortcomings:

- There is no guarantee that the risk premia thus estimated will be consistent with absence of arbitrage;
- No use is made of any information about the level of market yields: clearly, and estimate of, say, a −3% term premium has a different degree of ex ante plausibility depending on whether the corresponding market yield is, say, at 6% or 2%.

Traditionally, the 'other' route to estimating risk premia has been via the use of arbitrage-free affine term-structure models. Unfortunately, affine models do incorporate information about the level of market yields, and do ensure absence of arbitrage, but rarely do they have the flexibility to capture the rich information conveyed by the statistical analysis[4]. Both approaches are useful, but neither tells the whole truth.

The EDHEC Risk Premium Monitor exploits the relative strengths of the two approaches and tries to overcome their weaknesses. It does so by complementing the predictions from the statistical estimate with the assessment of the risk premium coming from a member of the family of affine models described in Rebonato (2017). The chosen model uses as state variables the short rate, its own stochastic reversion level and the market price of risk:[5]

$$dr_t^{\mathbb{Q}} = k_r^{\mathbb{P}}\left[\theta_t - r_t\right]dt + \sigma_r dz_t^r \quad (2)$$

$$d\theta_t^{\mathbb{Q}} = k_\theta^{\mathbb{P}}\left[\hat{\theta}_t^{\mathbb{P}} - \theta_t\right]dt + \lambda_t\sigma_\theta dt + \sigma_\theta dz_t^\theta \quad (3)$$

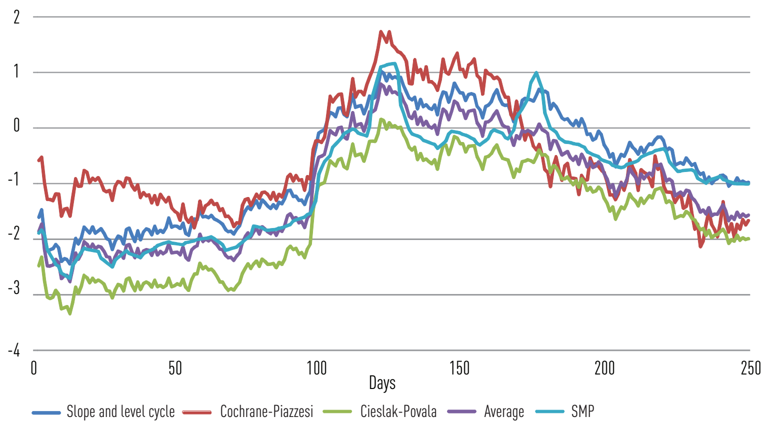
$$d\lambda_t = k_\lambda\left[\hat{\lambda}_t - \theta_t\right]dt + \sigma_\lambda dz_t^\lambda \quad (4)$$

where r_t, θ_t and λ_t are the time-t value of the short rate, of its instantaneous reversion level (the 'target rate') and the market price of risk, respectively; σ_r,σ_θ and σ_λ are the associated volatilities: $\hat{\theta}_t^{\mathbb{P}}$ and $\hat{\lambda}_t$ are the reversion levels of the 'target rate' and of the market price of risk, respectively; and the increments $dz^r{}_t$, $dz^\theta{}_t$ and $dz^\lambda{}_t$ suitably correlated. The model is fully specified once the initial state, r_0, θ_0 and λ_0 is given.

The reader is referred to Rebonato (2017) for the financial motivation of the model, and for a detailed description of its performance. For our purposes, the important observation is that the information about the P-measure path of the Fed funds (the 'short rate') comes from the forward guidance (the 'blue dots') provided quarterly by the Fed (see figure 6).

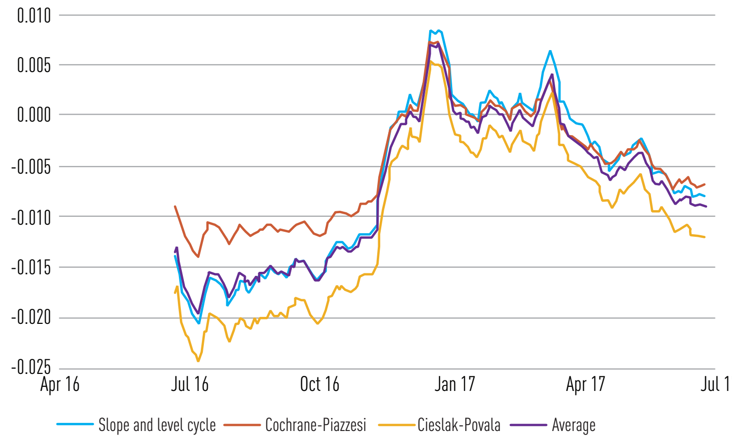
4 This is usually because the affine dependence of the market price of risk on the state variables (required to retain tractability) is too stylised to be quantitatively useful.
5 To simplify the analysis, and in line with standard findings (see, eg, Cochrane and Piazzesi [2005, 2008], Adrian, Crump and Moench [2013]), the model assumes that investors only seek compensation for the uncertainty about the level of rates, which we proxy in our approach as the long-term reversion level of the reversion level.

7. Estimates of the 10-year term premium (rebased)



Estimates from the statistical models and the affine model, rescaled to have the same volatility. The correlation among all the estimate is over 90%.

8. 10-year term premium



The result of combining the statistical estimates of the 10-year term premium from the models in the legend combined with the estimate from the affine model.

When the two sources of information are combined, we obtain for the 10-year term premium the composite robust estimates shown in figures 7 and 8. As figure 7 shows, the correlation among the statistical and the model-based estimate are above 90% for all the models. This is remarkable, considering how different the approaches and the sources of information are. This congruence gives us confidence about the robustness and the reliability of the combined approach.

Conclusions

In this article, we have given a glimpse of the latest and most exciting research strands carried out in the academic world in general, and at EDHEC-Risk Institute in particular, about the robust estimation of the yield risk premia. The predictions about the term premia for various yield maturities of the US Treasuries will be regularly provided, together with more formal research papers on these and related topics. Much work remains to be done, for instance by looking at different currencies, and at related asset classes. However, we believe that the present offering can already be of real practical use and interest for practitioners and for academics.

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